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Biderivations and Commuting Linear Maps on Current Lie Algebras

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Abstract. Let L be a Lie algebra and let A be an associative commutative algebra with unity, both over the same field F. We consider the following two questions. Is every skew-symmetric biderivation on the current Lie algebra $L \otimes A$ of the form $(x,y) \mapsto \lambda([x,y])$ for some $\gamma \in \mathrm{Cent}(L\otimes A)$, if the same holds true for L? Does every commuting linear map of $L\otimes A$ belong to $\mathrm{Cent}(L\otimes A)$, if the same holds true for L?

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1. Introduction

Let L be a Lie algebra over a field F. Recall that the centriod $\mathrm{Cent}(L)$ of L is defined as the space of all L-module endomorphisms of L, where L is viewed as an L-module under the adjoint action. This means that a linear map $\gamma : L \to L$ belongs to $\mathrm{Cent}(L)$ if and only if

$$\gamma([x,y]) = [x, \gamma(y)]$$

for all $x, y \in L$. The centralizer of a subset S of L is defined as

$$Z_L(S) = \{x \in L \mid [x,S] = 0\}.$$

Note that the center Z of L equals $Z_L(L)$. Next, by L' we denote the derived algebra $[L,L]$ of L.

A linear map $d : L \to L$ is called a *derivation* if $d([x,y]) = [d(x),y] + [x,d(y)]$ for all $x,y \in L$. A bilinear map $B : L\times L \to L$ is said to be a *skew-symmetric biderivation* if $B(x,y) = -B(y,x)$ for all $x,y \in L$ and

$$B([x,y],z) = [B(x,z),y] + [x, B(y,z)]$$

for all $x,y,z \in L$. In this case, $x \mapsto B(x,z)$ is a derivation for each $z \in L$. Obviously, $x \mapsto B(z,x)$ is also a derivation since B is skew-symmetric. Let us pick an arbitrary $\gamma \in \mathrm{Cent}(L)$. It turns out that the map $B : L \times L \to L$ defined by

$$B(x,y) = \lambda([x,y]) \tag{1}$$

is a skew-symmetric biderivation. A question that often appears in the study of biderivations is, when are all biderivations of the form (1).

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Recall that a linear map $f : L \to L$ satisfying $[f(x), x] = 0$ for all $x \in L$ is said to be a *commuting linear map*. It is well-known that commuting linear maps are closely related to skew-symmetric biderivations. Namely, if $f : L \to L$ is a commuting linear map it turns out that the map $B : L \times L \to L$ defined by

$$B(x, y) = [x, f(y)]$$

is a skew-symmetric biderivation. Obviously, each $\gamma \in \mathrm{Cent}(L)$ is a commuting linear map. This leads to a question of when do all commuting linear maps belong to $\mathrm{Cent}(L)$.

A systematic study of biderivations and commuting linear maps on associative rings and algebras was initiated in [1, 6]. For more information on this study we refer to the survey paper [2] and to the book on functional identities [5]. Recently, Brešar initiated the study of functional identities on tensor products of algebras [3, 4]. His results were inspiration for our previous paper [10], where we considered biderivations and commuting linear maps on the tensor product of two associative algebras. More recently, biderivations and commuting linear maps have been studied also in the context of Lie algebras [7, 8, 9, 12, 13, 14, 15, 16, 17]. In [7] Brešar and Zhao proved that for a perfect and centerless Lie algebra L every skew-symmetric biderivation $B : L \times L \to L$ is of the form (1). They also proved that for any Lie algebra L such that $Z_L(L') = 0$ every commuting linear map $f : L \to L$ belongs to $\mathrm{Cent}(L)$.

Let L and A be algebras over a field F, where L is a Lie algebra and A is an associative commutative algebra with unity. Then the tensor product algebra $L \otimes A$ is also a Lie algebra over F. Such Lie algebras are called *current Lie algebras*. Motivated by the above mentioned results from [7] and our previous paper [10] we consider the following two questions.

(a) Are all skew-symmetric biderivations on $L \otimes A$ of the form (1), if the same holds true for L?

(b) Does every commuting linear map of $L \otimes A$ belong to $\mathrm{Cent}(L \otimes A)$, if the same holds true for L?

In the second section we shall show that the answer to question (a) is positive if L is a prime Lie algebra or if $Z_L(L') = 0$ and A is finite dimensional. Using the above mentioned connection between skew-symmetric biderivations and commuting linear maps, one can conclude that the answer to question (b) is also positive in these two cases. However, in the third section we shall prove that the answer to question (b) is positive even under a milder assumption that L is centerless.

2. Biderivations

The basic idea is to compute $B([x, y], [u, v])$ in two different ways. This well-known idea was first used in [6], where biderivations of associative prime rings were studied. It was later used also in [9], where the following result was obtained.

Lemma 2.1. [9, Lemma 2.3] *Let L be a Lie algebra over a field F with* $\mathrm{char}(F) \neq 2$. *If $B : L \times L \to L$ is a skew-symmetric biderivation then*

$$[B(x, y), [u, v]] = [[x, y], B(u, v)] \qquad \textit{for all } x, y, u, v \in L.$$

Lemma 2.2. *Let $L \otimes A$ be a current Lie algebra over a field F with $\operatorname{char}(F) \neq 2$ and let $\{b_t \mid t \in T\}$ be a basis of A. Suppose that $B : (L \otimes A)^2 \to L \otimes A$ is a skew-symmetric biderivation. If $Z_L(L') = 0$ and each skew-symmetric biderivation on L is of the form (1), then there exists a family of endomorphisms $\{\gamma_t \mid t \in T\} \subseteq \operatorname{Cent}(L)$ such that for each pair $x, y \in L$ we have $\gamma_t([x,y]) = 0$ for all but finitely many $t \in T$ and*

$$B(x \otimes a, y \otimes b) = \sum_{t \in T} \gamma_t([x,y]) \otimes b_t ab \qquad \text{for all } x, y \in L \text{ and } a, b \in A.$$

Proof. For each pair $(x,y) \in (L \otimes A)^2$ there exist unique elements $B_t(x,y) \in L$, $t \in T$, such that

$$B(x,y) = \sum_{t \in T} B_t(x,y) \otimes b_t, \tag{2}$$

where $B_t(x,y) = 0$ for all but finitely many $t \in T$. Hence,

$$\begin{aligned}
\sum_{t \in T} B_t([x,y] \otimes 1, z \otimes 1) \otimes b_t &= B([x,y] \otimes 1, z \otimes 1) = B([x \otimes 1, y \otimes 1], z \otimes 1) \\
&= \big[B(x \otimes 1, z \otimes 1), y \otimes 1\big] - \big[B(y \otimes 1, z \otimes 1), x \otimes 1\big] \\
&= \Big[\sum_{t \in T} B_t(x \otimes 1, z \otimes 1) \otimes b_t, y \otimes 1\Big] - \Big[\sum_{t \in T} B_t(y \otimes 1, z \otimes 1) \otimes b_t, x \otimes 1\Big] \\
&= \sum_{t \in T} [B_t(x \otimes 1, z \otimes 1), y] \otimes b_t - \sum_{t \in T} [B_t(y \otimes 1, z \otimes 1), x] \otimes b_t
\end{aligned} \tag{3}$$

and so

$$\sum_{t \in T} \Big(B_t([x,y] \otimes 1, z \otimes 1) - [B_t(x \otimes 1, z \otimes 1), y] + [B_t(y \otimes 1, z \otimes 1), x]\Big) \otimes b_t = 0$$

for all $x, y, z \in L$. Consequently, for each $t \in T$ we have

$$B_t([x,y] \otimes 1, z \otimes 1) = [B_t(x \otimes 1, z \otimes 1), y] - [B_t(y \otimes 1, z \otimes 1), x]$$

for all $x, y, z \in L$. For each $t \in T$ we define a map $\overline{B}_t : L \times L \to L$ by $\overline{B}_t(x,y) = B_t(x \otimes 1, y \otimes 1)$. Then for every $t \in T$

$$\overline{B}_t([x,y], z) = [\overline{B}_t(x,z), y] - [\overline{B}_t(y,z), x]$$

for all $x, y, z \in L$. Thus, for each $t \in T$ the map $\overline{B}_t$ is a skew-symmetric biderivation on L and so the assumption implies that there exists $\gamma_t \in \operatorname{Cent}(L)$ such that

$$\overline{B}_t(x,y) = \gamma_t([x,y]) \tag{4}$$

for all $x, y \in L$. Thus, for each pair of elements $x, y \in L$ we know that $\gamma_t([x,y]) = 0$ for all but finitely many $t \in T$. Since L is centerless it is easy to see that γ_t is uniquely determined for each $t \in T$.

According to Lemma 2.1 we have

$$[B(x \otimes a, y \otimes b), [u \otimes 1, v \otimes 1]] = [[x \otimes a, y \otimes b], B(u \otimes 1, v \otimes 1)] \tag{5}$$

for all $x, y, u, v \in L$ and $a, b \in A$.

Using (2) and (4) we can rewrite (5) as

$$
\begin{aligned}
0 &= \Big[\sum_{t\in T} B_t(x\otimes a, y\otimes b)\otimes b_t, [u,v]\otimes 1\Big] + \Big[\sum_{t\in T}\gamma_t([u,v])\otimes b_t, [x,y]\otimes ab\Big] \qquad (6)\\
&= \sum_{t\in T}\Big[B_t(x\otimes a, y\otimes b), [u,v]\Big]\otimes b_t + \sum_{t\in T}\Big[\gamma_t([u,v]), [x,y]\Big]\otimes b_t ab\\
&= \sum_{t\in T}\Big[B_t(x\otimes a, y\otimes b), [u,v]\Big]\otimes b_t - \sum_{t\in T}\Big[\gamma_t([x,y]), [u,v]\Big]\otimes b_t ab
\end{aligned}
$$

for all $x, y, u, v \in L$ and $a, b \in A$. Obviously, for each $a \in A$ there exists a unique set of elements $\{\alpha_w(a) \in F \mid w \in T\}$ such that $a = \sum_{w\in T}\alpha_w(a)b_w$, where $\alpha_w(a) = 0$ for all but finitely many $w \in T$. Thus, for each $w \in T$ the map $a \mapsto \alpha_w(a)$ is well-defined. Consequently, (6) can be rewritten as

$$
\begin{aligned}
0 &= \sum_{w\in T}\big[B_w(x\otimes a, y\otimes b), [u,v]\big]\otimes b_w - \sum_{t\in T}\big[\gamma_t([x,y]), [u,v]\big]\otimes\big(\sum_{w\in T}\alpha_w(b_t ab)b_w\big)\\
&= \sum_{w\in T}\big[B_w(x\otimes a, y\otimes b), [u,v]\big]\otimes b_w - \sum_{t\in T}\sum_{w\in T}\alpha_w(b_t ab)\big[\gamma_t([x,y]), [u,v]\big]\otimes b_w\\
&= \sum_{w\in T}\big[B_w(x\otimes a, y\otimes b), [u,v]\big]\otimes b_w - \sum_{w\in T}\Big[\textstyle\sum_{t\in T}\alpha_w(b_t ab)\gamma_t([x,y]), [u,v]\Big]\otimes b_w\\
&= \sum_{w\in T}\Big(\big[B_w(x\otimes a, y\otimes b), [u,v]\big] - \big[\textstyle\sum_{t\in T}\alpha_w(b_t ab)\gamma_t([x,y]), [u,v]\big]\Big)\otimes b_w \qquad (7)
\end{aligned}
$$

and therefore for each $w \in T$ we have

$$
\begin{aligned}
0 &= \big[B_w(x\otimes a, y\otimes b), [u,v]\big] - \big[\textstyle\sum_{t\in T}\alpha_w(b_t ab)\gamma_t([x,y]), [u,v]\big]\\
&= \big[B_w(x\otimes a, y\otimes b) - \textstyle\sum_{t\in T}\alpha_w(b_t ab)\gamma_t([x,y]), [u,v]\big] \qquad (8)
\end{aligned}
$$

for all $x, y, u, v \in L$. Since $Z_L(L') = 0$ it follows that for each $w \in T$ we have

$$
B_w(x\otimes a, y\otimes b) = \sum_{t\in T}\alpha_w(b_t ab)\gamma_t([x,y]) \qquad (9)
$$

for all $x, y \in L$. Now, using (2) together with (9) we get for all $x, y \in L$ and $a, b \in A$

$$
\begin{aligned}
B(x\otimes a, y\otimes b) &= \sum_{w\in T} B_w(x\otimes a, y\otimes b)\otimes b_w\\
&= \sum_{w\in T}\big(\textstyle\sum_{t\in T}\alpha_w(b_t ab)\gamma_t([x,y])\big)\otimes b_w = \sum_{t\in T}\sum_{w\in T}\gamma_t([x,y])\otimes\alpha_w(b_t ab)b_w\\
&= \sum_{t\in T}\gamma_t([x,y])\otimes\big(\textstyle\sum_{w\in T}\alpha_w(b_t ab)b_w\big) = \sum_{t\in T}\gamma_t([x,y])\otimes b_t ab. \quad \blacksquare \qquad (10)
\end{aligned}
$$

Recall that a Lie algebra L is *prime*, if L has no nonzero ideals I, J such that $[I, J] = 0$. Let L be a prime Lie algebra. It is easy to see that $Z_L(L') = 0$ and so L is centerless. Moreover, it turns out that $\mathrm{Cent}(L)$ is a commutative integral domain and L is a torsion-free $\mathrm{Cent}(L)$-module (see [11, Theorem 1.1]). We are now ready to prove our main result on biderivations.

Theorem 2.3. *Let $L\otimes A$ be a current Lie algebra over a field F with* $\mathrm{char}(F)\neq 2$. *Assume that one of the following holds true:*

(i) *L is prime,*

(ii) *$Z_L(L')=0$ and $\dim_F(A)<\infty$.*

If each skew-symmetric biderivation on L is of the form (1)*, then the same holds true for $L\otimes A$.*

Proof. Let $B:(L\otimes A)^2\to L\otimes A$ be a skew-symmetric biderivation. Suppose that each skew-symmetric biderivation on L is of the form (1).

(i) First, we assume that L is a prime Lie algebra. Let $\{b_t \mid t\in T\}$ be a basis of A. According to Lemma 2.2 there exists a family of endomorphisms $\{\gamma_t \mid t\in T\}\subseteq \mathrm{Cent}(L)$ such that for each pair $x,y\in L$ we have $\gamma_t([x,y])=0$ for all but finitely many $t\in T$ and

$$B(x\otimes a, y\otimes b)=\sum_{t\in T}\gamma_t([x,y])\otimes b_t ab \tag{11}$$

for all $x,y\in L$ and $a,b\in A$. For each $t\in T$ we define a map $\lambda_t: L\otimes A\to L\otimes A$ by $\lambda_t=\gamma_t\otimes\mu_{b_t}$, where $\mu_{b_t}:A\to A$ is defined by $\mu_{b_t}(a)=b_t a$. It is easy to see that $\lambda_t\in\mathrm{Cent}(L\otimes A)$ for all $t\in T$. Obviously, we may assume that L is nonzero. Next, we claim that $\gamma_t=0$ for all but finitely many $t\in T$. Namely, since L is a nonzero prime Lie algebra there exist elements $x_0,y_0\in L$ such that $[x_0,y_0]\neq 0$. However, $\gamma_t([x_0,y_0])=0$ for all but finitely many $t\in T$. Since L is a prime Lie algebra, we know that $\mathrm{Cent}(L)$ is a commutative integral domain and L is a torsion-free $\mathrm{Cent}(L)$-module. Hence, it follows that $\gamma_t=0$ for all but finitely many $t\in T$. Consequently, $\lambda_t=0$ for all but finitely many $t\in T$. Let $\lambda=\sum_{t\in T}\lambda_t$. Obviously, $\lambda\in\mathrm{Cent}(L\otimes A)$ and (11) can now be written as

$$\begin{aligned}B(x\otimes a, y\otimes b)&=\sum_{t\in T}\gamma_t([x,y])\otimes b_t ab=\sum_{t\in T}(\gamma_t\otimes\mu_{b_t})([x,y]\otimes ab)\\&=\sum_{t\in T}\lambda_t([x\otimes a,y\otimes b])=\lambda([x\otimes a,y\otimes b])\end{aligned}\tag{12}$$

for all $x,y\in L$ and $a,b\in A$. Since B is bilinear and λ is linear it now follows that $B(x,y)=\lambda([x,y])$ for all tensors $x,y\in L\otimes A$.

(ii) We now assume that $Z_L(L')=0$ and $\dim_F(A)<\infty$. Let $\{b_1,\dots,b_n\}$ be a basis of A. According to Lemma 2.2 there exists a family of endomorphisms $\{\gamma_1,\dots,\gamma_n\}\subseteq\mathrm{Cent}(L)$ such that

$$B(x\otimes a, y\otimes b)=\sum_{t=1}^{n}\gamma_t([x,y])\otimes b_t ab \tag{13}$$

for all $x,y\in L$ and $a,b\in A$. For each $t\in\{1,2,\dots,n\}$ we define a map $\lambda_t: L\otimes A\to L\otimes A$ by $\lambda_t=\gamma_t\otimes\mu_{b_t}$. It is easy to see that $\lambda_t\in\mathrm{Cent}(L\otimes A)$ for all $t\in\{1,2,\dots,n\}$. Let $\lambda=\sum_{t=1}^{n}\lambda_t$. Obviously, $\lambda\in\mathrm{Cent}(L\otimes A)$ and (13) can now be written as

$$\begin{aligned}B(x\otimes a, y\otimes b)&=\sum_{t=1}^{n}\gamma_t([x,y])\otimes b_t ab=\sum_{t=1}^{n}(\gamma_t\otimes\mu_{b_t})([x,y]\otimes ab)\\&=\sum_{t=1}^{n}\lambda_t([x\otimes a,y\otimes b])=\lambda([x\otimes a,y\otimes b])\end{aligned}\tag{14}$$

for all $x, y \in L$ and $a, b \in A$. Since B is bilinear and λ is linear it now follows that $B(x,y) = \lambda([x,y])$ for all $x, y \in L \otimes A$. ■

3. Commuting linear maps

Recall that each $\gamma \in \mathrm{Cent}(L)$ is a commuting linear map since $[\gamma(x), x] = 0$ for all $x \in L$. Even for centerless Lie algebras the converse is not true (see [7, Example 3.4]). In this section we consider question (b). Does every commuting linear map of $L \otimes A$ belong to $\mathrm{Cent}(L \otimes A)$, if the same holds true for L? Using Theorem 2.3 and the well-known connection between skew-symmetric biderivations and commuting linear maps one can show that the answer to question (b) is positive if L is a prime Lie algebra or if $Z_L(L') = 0$ and A is finite dimensional. However, using a more direct approach, we are able to obtain the following theorem, which states that the answer to question (b) is positive even under a milder assumption that L is centerless.

Theorem 3.1. *Let $L \otimes A$ be a current Lie algebra over a field F with* $\mathrm{char}(F) \neq 2$, *where L is a centerless Lie algebra. If each commuting linear map of L belongs to* $\mathrm{Cent}(L)$, *then the same holds true for $L \otimes A$.*

Proof. Let $f : L \otimes A \to L \otimes A$ be a commuting linear map and let $\{b_t \mid t \in T\}$ be a basis of A. For any tensor $x \in L \otimes A$ there exist unique elements $f_t(x) \in L$, $t \in T$, such that

$$f(x) = \sum_{t \in T} f_t(x) \otimes b_t, \tag{15}$$

where $f_t(x) = 0$ for all but finitely many $t \in T$. Hence,

$$0 = [f(x \otimes 1), x \otimes 1] = \Big[\sum_{t \in T} f_t(x \otimes 1) \otimes b_t, x \otimes 1\Big] = \sum_{t \in T} [f_t(x \otimes 1), x] \otimes b_t$$

for any $x \in L$. Consequently, for each $x \in L$ we have $[f_t(x \otimes 1), x] = 0$ for all $t \in T$. For each $t \in T$ we define a map $\overline{f}_t : L \to L$ by $\overline{f}_t(x) = f_t(x \otimes 1)$. Obviously, $\overline{f}_t$ is a commuting linear map and so the assumption implies that $\overline{f}_t \in \mathrm{Cent}(L)$ for all $t \in T$. On the other hand, f is a commuting linear map on $L \otimes A$ and hence

$$\begin{aligned} 0 &= [f(x \otimes a + y \otimes 1), x \otimes a + y \otimes 1] \\ &= [f(x \otimes a), y \otimes 1] + [f(y \otimes 1), x \otimes a] \\ &= \Big[\sum_{t \in T} f_t(x \otimes a) \otimes b_t, y \otimes 1\Big] + \Big[\sum_{t \in T} f_t(y \otimes 1) \otimes b_t, x \otimes a\Big] \\ &= \sum_{t \in T} [f_t(x \otimes a), y] \otimes b_t + \sum_{t \in T} [f_t(y \otimes 1), x] \otimes b_t a \end{aligned}$$

for all $x, y \in L, a \in A$. Thus,

$$\sum_{t \in T} [f_t(x \otimes a), y] \otimes b_t + \sum_{t \in T} [\overline{f}_t(y), x] \otimes b_t a = 0 \tag{16}$$

for all $x, y \in L, a \in A$.

Obviously, for each $a \in A$ there exists a unique set of elements $\{\alpha_w(a) \in F \mid w \in T\}$ such that $a = \sum_{w\in T} \alpha_w(a) b_w$, where $\alpha_w(a) = 0$ for all but finitely many $w \in T$. Thus, for each $w \in T$ the map $a \mapsto \alpha_w(a)$ is well-defined. Consequently, (16) can be rewritten as

$$\begin{aligned} 0 &= \sum_{t\in T} [f_t(x \otimes a), y] \otimes b_t + \sum_{t\in T} [\overline{f}_t(y), x] \otimes \left(\textstyle\sum_{w\in T} \alpha_w(b_t a) b_w \right) \\ &= \sum_{w\in T} [f_w(x \otimes a), y] \otimes b_w + \sum_{t\in T} \sum_{w\in T} \alpha_w(b_t a) [\overline{f}_t(y), x] \otimes b_w \\ &= \sum_{w\in T} [f_w(x \otimes a), y] \otimes b_w + \sum_{w\in T} \left[y, \textstyle\sum_{t\in T} \alpha_w(b_t a) \overline{f}_t(x) \right] \otimes b_w \\ &= \sum_{w\in T} \left[f_w(x \otimes a) - \textstyle\sum_{t\in T} \alpha_w(b_t a) \overline{f}_t(x), y \right] \otimes b_w \end{aligned} \tag{17}$$

and therefore for each $w \in T$ we have

$$\left[f_w(x \otimes a) - \textstyle\sum_{t\in T} \alpha_w(b_t a) \overline{f}_t(x), y \right] = 0$$

for all $x, y \in L$. Since L is centerless it follows that

$$f_w(x \otimes a) = \sum_{t\in T} \alpha_w(b_t a) \overline{f}_t(x) \tag{18}$$

for all $x \in L$ and $w \in T$. Note that for each $x \in L$ we have $\overline{f}_t(x) = 0$ for all but finitely many $t \in T$. Using (15) and (18) we obtain

$$\begin{aligned} f(x \otimes a) &= \sum_{w\in T} f_w(x \otimes a) \otimes b_w = \sum_{w\in T} \Big(\textstyle\sum_{t\in T} \alpha_w(b_t a) \overline{f}_t(x) \Big) \otimes b_w \\ &= \sum_{t\in T} \sum_{w\in T} \alpha_w(b_t a) \overline{f}_t(x) \otimes b_w = \sum_{t\in T} \overline{f}_t(x) \otimes \Big(\textstyle\sum_{w\in T} \alpha_w(b_t a) b_w \Big) \\ &= \sum_{t\in T} \overline{f}_t(x) \otimes b_t a \end{aligned}$$

for all $x \in L$ and $a \in A$. Consequently,

$$\begin{aligned} f([x \otimes a, y \otimes b]) = f([x, y] \otimes ab) &= \sum_{t\in T} \overline{f}_t([x, y]) \otimes b_t ab \\ &= \sum_{t\in T} [x, \overline{f}_t(y)] \otimes a b_t b = \sum_{t\in T} [x \otimes a, \overline{f}_t(y) \otimes b_t b] \\ &= \left[x \otimes a, \sum_{t\in T} \overline{f}_t(y) \otimes b_t b \right] = [x \otimes a, f(y \otimes b)] \end{aligned}$$

for all $x, y \in L$ and $a, b \in A$. Since f is a linear map we obtain directly that $f([x, y]) = [x, f(y)]$ for all tensors $x, y \in L \otimes A$. Thus, $f \in \mathrm{Cent}(L \otimes A)$. ∎

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