Erratum to "Trace class groups"

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Abstract. We show that a reductive Lie group is trace class and thus correct an error in a former paper [A. Deitmar, G. van Dijk, *Trace class groups*, J. Lie Theory 26 (2016) 269–291]. We reduce to the semisimple case, which was done by Harish-Chandra.

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As was brought to the author's attention by Karl-Hermann Neeb, the proof of Theorem 2.1 of [1] of the author together with Gerrit van Dijk[1] contains an error: the implicit assumption was made, that the central character would have compact image. Below we correct this by showing how to reduce the claim to the semisimple case which has been treated by Harish-Chandra.

Definiton 1.1. By a *semisimple*, resp. *reductive* Lie group we mean a Lie group with semisimple resp. reductive Lie algebra.

Theorem 1.2 *A connected reductive Lie group is trace class.*

Proof. Let G be a connected reductive Lie group and (π, V_π) an irreducible unitary representation of G. Let Z be the center of G and $H = G_{\mathrm{der}}$ its derived group. Multiplication gives a surjective homomorphism $H \times Z \to G$ with kernel $N = H \cap Z$. For given $f \in C_c^\infty(G)$ there exists $g \in C_c^\infty(H \times Z)$ with

$$f(x) = \int_N g(nx) \, dn.$$

Replacing f with g and π with its pullback to $H \times Z$ we may assume that $G = H \times Z$. Note that the restriction of π to H is still irreducible.

The central group Z acts on V_π by a unitary character $\chi_\pi : Z \to \mathbb{C}^\times$. The function

$$f_\pi(x) = \int_Z f(xz)\chi_\pi(z) \, dz$$

lies in $C_c^\infty(H)$ and satisfies $\pi|_H(f_\pi) = \pi(f)$. We therefore can replace f with f_π and G with H, i.e., we can assume G to be semisimple.

The semisimple case has been dealt with by Harish-Chandra, see [2] p. 244, line 6. Alternatively, one can look at Theorem 2 in [2] and use Proposition 1.6 of [1]. ∎

[1] deceased in 2022.

References

[1]   A. Deitmar, G. van Dijk: *Trace class groups*, J. Lie Theory 26/1 (2016) 269–291.

[2]   Harish-Chandra: *Representations of semisimple Lie groups III*, Trans. Amer. Math. Soc. 76 (1954) 234–253.

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