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Erratum to "From Euler Elements and 3-Gradings to Non-Compactly Causal Symmetric Spaces", Journal of Lie Theory 33 (2023) 377–432

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Abstract. This is an erratum to our previous paper "From Euler Elements and 3-Gradings to Non-Compactly Causal Symmetric Spaces", Journal of Lie Theory 33 (2023) 377–432.

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Theorem 7.8 of our article asserts in particular that, for the Lie algebra $\mathfrak{so}_{2r,2r}(\mathbb{R})$, an Euler element $h \in \mathfrak{g}$ of split type, and the corresponding connected centerfree group $G = \mathrm{SO}_{2r,2r}(\mathbb{R})_e/\{\pm\mathbf{1}\}$, the centralizer

$$G^h = \{g \in G \colon \mathrm{Ad}(g)h = h\}$$

has two connected components. This statement is false and the group G^h is connected.

This can be seen as follows. We realize $\mathfrak{g} = \mathfrak{so}_{2r,2r}(\mathbb{R})$ as the matrix Lie algebra corresponding to the symmetric bilinear form on $\mathbb{R}^{4r}$, represented by the symmetric block matrix $\beta := \begin{pmatrix} 0 & \mathbf{1}_{2r} \\ \mathbf{1}_{2r} & 0 \end{pmatrix}$:

$$\mathfrak{so}_{2r,2r}(\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & a^{-\top} \end{pmatrix} : a \in \mathfrak{gl}_{2r}(\mathbb{R}),\ b, c \in \mathrm{Skew}_{2r}(\mathbb{R}) \right\}.$$

We consider the diagonal Euler element

$$h = \frac{1}{2} \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}.$$

Its centralizer is

$$\mathrm{SO}_{2r,2r}(\mathbb{R})^h = \left\{ \begin{pmatrix} g & 0 \\ 0 & g^{-\top} \end{pmatrix} : g \in \mathrm{GL}_{2r}(\mathbb{R}) \right\} \cong \mathrm{GL}_{2r}(\mathbb{R}),$$

acting on $\mathrm{Skew}_{2r}(\mathbb{R})$ by $g.A = gAg^\top$.

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The maximal compact subgroup of $\mathrm{SO}_{2r,2r}(\mathbb{R})_e$ is isomorphic to the connected group

$$\mathrm{SO}_{2r}(\mathbb{R}) \times \mathrm{SO}_{2r}(\mathbb{R}),$$

where the two components of $k = (k_1, k_2)$, $k_j \in \mathrm{SO}_{2r}(\mathbb{R})$, act on the two eigenspaces of β. The corresponding matrices take in our coordinates the block form

$$k = \frac{1}{2}\begin{pmatrix} k_1 + k_2 & k_1 - k_2 \\ k_1 - k_2 & k_1 + k_2 \end{pmatrix} \in \mathrm{O}_{4r}(\mathbb{R}) \cap \mathrm{SO}_{2r,2r}(\mathbb{R}).$$

Therefore the maximal compact subgroup of $(\mathrm{SO}_{2r,2r}(\mathbb{R})_e)^h$ corresponds to the pairs with $k_1 = k_2$, hence is isomorphic to the connected group $\mathrm{SO}_{2r}(\mathbb{R})$. Therefore polar decomposition shows that the maximal compact subgroups of $(\mathrm{SO}_{2r,2r}(\mathbb{R})_e)^h$ are isomorphic to $\mathrm{SO}_{2r}(\mathbb{R})$, hence connected. We conclude that $(\mathrm{SO}_{2r,2r}(\mathbb{R})_e)^h$ is connected, and this implies that its adjoint image G^h is connected.

We take the chance to remark that two minor typos survived in the printed version of our article:

- In line 4 of the proof of Theorem 7.8, $-\alpha$ should be replaced by $-\tau$, and
- and in line 20, BC_r should be replaced by C_r.

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