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Erratum to "First Extension Groups of Verma Modules and *R*-Polynomials"

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Abstract. This is a correction to a former paper of the author [*First extension groups of Verma modules and R-polynomials*, J. Lie Theory 25/2 (2015) 377–393].

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Theorem 1.2 in [1] is false. We can only prove the inequality, namely we only have the following theorem.

Theorem 1. *Assume that λ is regular. Then $\dim V_\lambda(x,y)$ is less than or equal to the coefficient of q in $(-1)^{\ell(y)-\ell(x)-1}R_{y,x}(q)$.*

When $x = w_0$, we have the equality.

Theorem 2. *Assume that λ is regular. Then $\dim V_\lambda(w_0,x)$ is equal to the coefficient of q in $(-1)^{\ell(x)-\ell(w_0)-1}R_{x,w_0}(q)$.*

Both theorems follow from the proof of [1, Theorem 4.4]. In the proof of Theorem [1, Theorem 4.4], we claimed that [1, Theorem 4.4] follows from Theorem 2. However this argument is not correct.

Here is a counterexample of [1, Theorem 4.4]. We use the notation in the proof of [1, Theorem 4.4]. Let $\mathfrak{g}$ be the simple Lie algebra of type B_3 and we use the standard notation of the root system. In particular, the set of simple roots is $\{e_1 - e_2, e_2 - e_3, e_3\}$. Let $s_1 = s_{e_1-e_2}, s_2 = s_{e_2-e_3}, s_3 = s_{e_3}$ be simple reflections. Put $x = s_2s_3s_2s_1s_2s_3$ and $y = s_3s_2$. Then using the formula in the proof of [1, Theorem 4.4], we have

$$\begin{aligned} r_{s_2s_3s_2s_1s_2s_3,s_3s_2} &= r_{s_2s_3s_2s_1s_2,s_3s_2} + 1 \\ &= r_{s_2s_3s_2s_1,s_3} + 1 \\ &= r_{s_2s_3s_2,s_3} + 2 \\ &= r_{s_2s_3,s_3} + 3 \\ &= r_{s_2,e} + 3 \\ &= r_{e,e} + 4 = 4. \end{aligned}$$

However $n_{x,y} \le \dim \mathfrak{h} = 3$. Hence $n_{x,y} < r_{x,y}$.

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References

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