Foreword

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Abstract.   This special issue is a tribute to the journal's long time managing editor Karl-Hermann Neeb on the occasion of his 60th birthday.

Karl-Hermann Neeb was born on February 26, 1964. He started his study of Mathematics at the TH Darmstadt in 1983. Soon he became a member of the research group of K. H. Hofmann, which at that time was leading in research on invariant cones in Lie algebras. Karl-Hermann earned his doctorate under the supervision of K. H. Hofmann at the TH Darmstadt in 1990. His dissertation, titled *Globality of Lie Wedges*, laid the groundwork for his lifelong engagement with invariant cones, positivity and holomorphy in representation theory, convexity, structure of Lie groups and homogeneous spaces, and applications in related fields.

Karl-Hermann obtained a Postdoctoral Fellowship from the DFG after his graduation and visited Université Pierre et Marie Curie in Paris during the academic year 1990–1991. Here he worked with J. Faraut and his research group, getting familiar with Jordan algebras and their applications in analysis and representation theory. This toolbox has become a recurring theme in many of Karl-Hermann's articles.

The next step was a position as assistant professor at the TH Darmstadt in 1991. Three years later, he became associate professor at the FAU Erlangen-Nürnberg in 1994. His first appointment as full professor was at the TH Darmstadt in 1998. Finally, in 2010, he returned to the FAU Erlangen-Nürnberg as full professor, and has held this position since.

Karl-Hermann has over the years had nineteen graduate students, ten in Darmstadt and nine in Erlangen. Two of them, H. Glöckner and B. Krötz, both full professors at the Department of Mathematics in Paderborn, have become leading researchers in their fields and supervised in total ten graduate students themselves.

According to MathSciNet, as of October 8, 2025, Karl-Hermann has authored or co-authored an impressive amount of 188 research articles, 8 survey papers, 6 books and collections, 4 research monographs, 35 book chapters and 2 textbooks. He has at this point 57 coauthors, among them G. Ólafsson with 23 joint publications and J. Hilgert with 22. His most cited work includes his self-contained reference book on invariant cones, positivity, convexity and holomorphic representations [34], his textbook with J. Hilgert on the structure and geometry of Lie groups [16], his monograph on infinite-dimensional Lie groups [38], which is one of the standard

references on the subject, and finally the Lecture Notes with J. Hilgert on semigroups in Lie groups and their applications in geometry and analysis [14], containing a very accessible description of the status of the research in representation theory, ordered spaces and Hardy spaces at that point.

Karl-Hermann is without doubt a leading force in the theory of positivity and convexity in representation theory and geometry as well as infinite-dimensional Lie groups.

He started out with order structures in Lie groups and Lie algebras, studying in particular their relation with representation theory in the context of holomorphic discrete series discovered by Ol'shanskii [51, 52]. This made him study Lie theory, differential and complex geometry as well as the functional analysis of representation theory, which have been recurring themes of his research ever since.

He extended the groundbreaking work of Harish-Chandra on highest weight representations [12, 13] to the most general settings of groups whose Lie algebra contains convex cones with non-empty interior. This theory also extended the work of Ol'shanskii [51, 52] on subsemigroups of Lie groups related to holomorphic representations of semisimple Lie groups to the setting of involutive semigroups. It can be specialized to Ol'shanskii semigroups which are semigroups of the form $S(C) = H \exp iC$ with involution $s^* = \tau(s)^{-1}$ where $\tau : G \to G$ is an involution and H is a symmetric subgroup corresponding to τ and $C \subset \mathfrak{g}^{-d\tau}$ is a pointed generating hyperbolic H-invariant cone studied in [26]. In the group case, τ is the integrated conjugation with respect to $\mathfrak{g}$.

On the way, he proved a convexity theorem for ordered symmetric spaces G/H and Ol'shankii semigroups, see [20, 32] and [34, Thm. VIII.1.36], and studied moment maps and convex momentum sets for unitary representations. All of this was collected in the self-contained book [34] where the theory was built up from the beginning and presented in all generalities. This book quickly became a standard reference for the theory and still is.

Irreducible unitary highest weight representations of semisimple Lie groups were first classified by T. Enright, R. Howe and N. Wallach [7] and H. P. Jakobsen [21]. The construction was algebraic and it has been an open problem to give an analytic description of the unitary structure. It was a natural step for Karl-Hermann and his co-authors to take a look at this problem, which led to a series of three publications [17, 18, 19] about an analytic construction of unitary highest weight representations via the orbit method. Those articles deal with the finite-dimensional case, but Karl-Hermann also worked on the much harder problem of constructing unitary highest weight representations of infinite-dimensional groups [22, 23, 27, 33, 37, 39, 47, 49].

Infinite-dimensional Lie groups are common in mathematics and physics in the form of mapping groups, gauge groups, injective and projective limits of Lie groups and as a tool in the solution of differential equations which often can be expressed in terms of integral curves of time-dependent invariant vector fields. Karl-Hermann has made several contributions to the theory of infinite-dimensional Lie groups and topological groups and worked on related problems for most of his research career. This includes representation theory as mentioned earlier but also the structure theory of those groups and their Lie algebras, as well as work on special classes

of groups and Lie algebras. The basic structure theory of those groups is far from well understood, and several results that are well known for finite-dimensional groups and Lie algebras still remain unsolved in infinite dimension. The simplest cases are where the model space is a Hilbert or Banach space, but problems become more complicated if the model vector space is just assumed locally convex and complete. Some of those open problems are discussed in [9, 38] and include topics like: When is the exponential map a local homeomorphism? Which Lie sub-algebras correspond to analytic subgroups? Those questions are closely related and discussed in [38, Chap. 4]. Another question in this context that Karl-Hermann has worked on is that of central extensions for infinite-dimensional groups [36], which is discussed extensively in [38, Chap. 5]. This work is essential for modern theoretical physics and functional analysis. Further contributions to the theory of infinite-dimensional groups and their representations are [5, 10, 33, 35, 37, 40, 41, 48, 50].

While in the background of his work early on, for instance in [15], which was motivated by index theory for families of operators on ordered structures, C^*-algebras showed up in Karl-Hermann's research repeatedly. Sometimes in the guise of examples such as the invertible elements of C^*-algebras as in [2], sometimes in the attempt to extend methods from the finite-dimensional to the infinite-dimensional context as in [50]. During the last fifteen years when Karl-Hermann started to focus on questions related to mathematical physics, C^*-algebraic methods played an important role in many of his papers, e.g in his collaboration with H. Grundling [11]. But most importantly they feature in his more recent work on Algebraic Quantum Field Theory in the spirit of Haag-Kastler, see e.g. [42, 46]. As this is at the center of his current research and incorporates almost all topics he touched on during his research career, we go a little more into detail for this subject.

It started with reflection positivity [1, 25, 43, 45], moving a unitary representation from one real form of a complex Lie group to another, also referred to as "Wick rotation" in the physics literature. The process is carried out using the Lie algebra action, which immediately leads to questions concerning the globalisation of representations of a Lie algebra [24, 28, 29]. But soon the project changed direction towards the Tomita–Takesaki Theorem [6, Thm. 2.5.14] (a key result in C^*-algebra theory), standard subspaces and geometry of symmetric spaces, see [30, 31]. On the Lie algebra level, the main objects are *Euler elements* $h \in \mathfrak{g}$, i.e., elements such that $\operatorname{spec}(\operatorname{ad} h) \subset \{-1, 0, 1\}$ and hence defining a three grading $\mathfrak{g} = \mathfrak{g}_{-1} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_1$. (Note that one of the spaces $\mathfrak{g}_{\pm 1}$ might be trivial.) Furthermore, h defines an involution $\tau_h^{\mathfrak{g}} = e^{\pi i \operatorname{ad}(h)} : \mathfrak{g} \to \mathfrak{g}$ and one assumes that it integrates to an involution $\tau_h : G \to G$. The extended symmetry group is the semidirect product $G_{\tau_h} = G \rtimes \{1, \tau_h\}$, where one can think of τ_h as the time reflection. One then considers *anti-unitary* representations of G_{τ_h}, i.e., representations that are unitary on G while $U(\tau_h) = J$ is anti-unitary [44]. Then $JU(g)J = U(\tau_h(g))$ and the pair (Δ, J) with $\Delta = e^{2\pi i \partial U(h)}$ satisfies the Tomita-Takesaki relations.

On the way, several deep results in representation were established. One of them is the realisation of representations in spaces of distributions using a boundary value map along curves in the complex crown with limit points on the symmetric space G/H realized as the boundary of the crown [8]. Those ideas led to the concept of crown domains in arbitrary Lie groups, holomorphic extension and limits in the space of distribution vectors [3, 4].

Finally, Karl-Hermann's service to the mathematical community is just as noticable as his research contributions, in particular his responsibilities as editor. He played an important role in the Journal of Lie Theory from the very beginnings of this journal. In fact, he has a paper in Volume 1, Number 1 (1991), when it was still published as Seminar Sophus Lie. As an assistant to the first managing editor K. H. Hofmann, he was constantly involved in matters of the journal, and he kept up his routine as a reviewer and author (seven papers in Seminar Sophus Lie and five papers in the Journal of Lie Theory) when he moved on to his first tenured position in Erlangen. When he succeeded his former teacher on the Chair of Functional Analysis in Darmstadt, he also took on the challenge as a managing editor of the Journal of Lie Theory. He kept this position for 25 years, and is still serving as deputy managing editor helping everybody with his immense experience.

Besides his involvement with the Journal of Lie Theory, Karl-Hermann has served as an editor for a number of other journals as well as proceedings volumes. Moreover, for nearly three decades he has had the informal leadership of the Seminar Sophus Lie which ever since 1991 meets at least once a year somewhere in Europe. On the department level, he has served as a chair of the Department of Mathematics in Erlangen twice, 2013–2015 and 2023–2025. Moreover, he was a member of the steering committee of the DFG Priority Programme 1388: Representation theory (2008–2015).

All this bears witness to Karl-Hermann's deep love for mathematics and his willingness to share his fundamental insights, paired with a friendly personality and a remarkable lack of arrogance.

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References

[1] M. S. Adamo, K.-H. Neeb, J. Schober: *Reflection positivity and its relation to disc, half plane and the strip*, Expo. Math. 43/4 (2025), art.no. 125660.

[2] D. Beltiţă, K.-H. Neeb: *Schur-Weyl theory for C^*-algebras*, Math. Nachr. 285 (2012) 1170–1198.

[3] D. Beltiţă, K.-H. Neeb: *Holomorphic extension of one-parameter operator groups*, Pure Appl. Funct. Anal. 9 (2024) 1483–1526.

[4] D. Beltiţă, K.-H. Neeb: *Crowned Lie groups and nets of real subspaces*, arXiv: 2506. 16422 (2025).

[5] W. Bertram, H. Glöckner, K.-H. Neeb: *Differential calculus over general base fields and rings*, Expo. Math. 22 (2004) 213–282.

[6] O. Bratteli, D. W. Robinson: *Operator Algebras and Quantum Statistical Mechanics I*, 2nd ed., Texts and Monographs in Physics, Springer, Berlin (1987).

[7] T. Enright, R. Howe, N. Wallach: *A classification of unitary highest weight modules*, in: *Representation Theory of Reductive Groups*, Park City, 1982, Progress in Mathematics vol. 40, Birkhäuser, Boston (1983) 97–143.

[8]   J. Frahm, K.-H. Neeb, G. Ólafsson: *Nets of standard subspaces on non-compactly causal symmetric spaces*, in: Symmetry in Geometry and Analysis, Vol. 2. Festschrift in honor of Toshiyuki Kobayashi, Progress in Mathematics vol. 358, Birkhäuser/Springer, Singapore (2025) 115–195.

[9]   H. Glöckner: *Fundamental problems in the theory of infinite-dimensional Lie groups*, J. Geom. Symmetry Phys. 5 (2006) 24–35.

[10] H. Glöckner, K.-H. Neeb: *Infinite Dimensional Lie Groups*, book in preparation.

[11] H. Grundling, K.-H. Neeb: *Crossed products of C^*-algebras for singular actions with spectrum conditions*, J. Operator Theory 84 (2020) 369–451.

[12] Harish-Chandra: *Representations of semisimple Lie groups, IV*, Amer. J. Math. 77 (1955) 743–777.

[13] Harish-Chandra: *Representations of semisimple Lie groups, V*, Amer. J. Math. 78 (1956) 1–41.

[14] J. Hilgert, K.-H. Neeb: *Lie Semigroups and their Applications*, Lecture Notes in Mathematics 1552, Springer, Berlin (1993).

[15] J. Hilgert, K.-H. Neeb: *Wiener-Hopf operators on ordered homogeneous spaces. I*, J. Funct. Anal. 132 (1995) 86–118.

[16] J. Hilgert, K.-H. Neeb: *Structure and Geometry of Lie Groups*, Springer Monographs in Mathematics, Springer, New York (2012).

[17] J. Hilgert, K.-H. Neeb, B. Ørsted: *The geometry of nilpotent coadjoint orbits of convex type in Hermitian Lie algebras*, J. Lie Theory 4 (1994) 185–235.

[18] J. Hilgert, K.-H. Neeb, B. Ørsted: *Conal Heisenberg algebras and associated Hilbert spaces*, J. Reine Angew. Math. 474 (1996) 67–112.

[19] J. Hilgert, K.-H. Neeb, B. Ørsted: *Unitary highest weight representations via the orbit method. I: The scalar case*, Acta Appl. Math. 44 (1996) 151–184.

[20] J. Hilgert, K.-H. Neeb, W. Plank: *Symplectic convexity theorems and coadjoint orbits*, Compositio Math. 96a (1994) 129–180.

[21] H. P. Jakobsen: *Hermitian symmetric spaces and their unitary highest weight modules*, J. Funct. Anal. 52 (1983) 385–412.

[22] B. Janssens, K.-H. Neeb: *Projective unitary representations of infinite-dimensional Lie groups*, Kyoto J. Math. 59 (2019) 293–341.

[23] B. Janssens, K.-H. Neeb: *Positive Energy Representations of Gauge Groups. I: Localization*, Memoirs of the EMS 9, European Mathematical Society, Berlin (2024).

[24] P. E. T. Jorgensen: *Analytic continuation of local representations of Lie groups*, Pacific J. Math. 125 (1986) 397–408.

[25] P. E. T. Jorgensen, K.-H. Neeb, G. Ólafsson: *Reflection positive stochastic processes indexed by Lie groups*, SIGMA Symmetry Integrability Geom. Methods Appl. 12 (2016), art. no. 58, 49 pp.

[26] B. Krötz, K.-H. Neeb: *On hyperbolic cones and mixed symmetric spaces*, J. Lie Theory 6 (1996) 69–146.

[27] T. Marquis, K.-N. Neeb: *Positive energy representations for locally finite split Lie algebras*, Int. Math. Res. Notices 2016/21 (2016) 6689–6712.

[28] S. Merigon, K.-H. Neeb: *Analytic extension techniques for unitary representations of Banach-Lie groups*, Int. Math. Res. Notices 18 (2012) 4260–4300.

[29] S. Merigon, K.-H. Neeb, G. Ólafsson: *Integrability of unitary representations on reproducing kernel spaces*, Represent. Theory 19 (2015) 24–55.

[30] V. Morinelli, K.-H. Neeb: *Covariant homogeneous nets of standard subspaces*, Comm. Math. Phys. 386 (2021) 305–358.

[31] V. Morinelli, K.-H. Neeb: *From local nets to Euler elements*, Adv. Math. 458 (2024), art.no. 109960.

[32] K.-H. Neeb: *A convexity theorem for semisimple symmetric space*, Pac. J. Math. 162 (1994) 305–349.

[33] K.-H. Neeb: *Holomorphic highest weight representations of infinite-dimensional complex classical groups*, J. Reine Angew. Math. 497 (1998) 171–222.

[34] K.-H. Neeb: *Holomorphy and Convexity in Lie Theory*, De Gruyter Expositions in Mathematics vol. 28, Walter De Gruyter & Co., Berlin (2000).

[35] K.-H. Neeb: *A Cartan-Hadamard theorem for Banach-Finsler manifolds*, in: Proc. Conf. Geometric and Combinatorial Group Theory, Part II, Haifa 2000, Geom. Dedicata 95 (2002) 115–156.

[36] K.-H. Neeb: *Central extensions of infinite-dimensional Lie groups*, Ann. Inst. Fourier (Grenoble) 52 (2002) 1365–1442.

[37] K.-H. Neeb: *Infinite-dimensional groups and their representations*, in: Lie Theory, Lie Algebras and Representations, J.-P. Anker et al. (eds.), Progress in Mathematics vol. 228, Birkhäuser, Boston (2004) 213–328.

[38] K.-H. Neeb: *Towards a Lie theory of locally convex groups*, Jap. J. Math. 1 (2006) 291–468.

[39] K.-H. Neeb: *Unitary highest weight modules of locally affine Lie algebras*, Contemp. Math. 506 (2010) 227–262.

[40] K.-H. Neeb: *On differentiable vectors for representations of infinite dimensional Lie groups*, J. Funct. Anal. 259 (2010) 2814–2855.

[41] K.-H. Neeb: *On analytic vectors for unitary representations of infinite dimensional Lie groups*, Ann. Inst. Fourier (Grenoble) 61 (2011) 1839–1874.

[42] K.-H. Neeb: *Nets of real subspaces on homogeneous spaces and algebraic quantum field theory*, to appear in: Proc. of the Trimester "Representation Theory and Noncommutative Geometry" at Institut Henri Poincaré in 2025, Progress in Mathematics, Birkhäuser, Basel (2025).

[43] K.-H. Neeb, G. Ólafsson: *Reflection positivity and conformal symmetry*, J. Funct. Anal. 266 (2014) 2174–2224.

[44] K.-H. Neeb, G. Ólafsson: *Antiunitary representations and modular theory*, Banach Center Publ. 113 (2017) 291–362.

[45] K.-H. Neeb, G. Ólafsson: *Reflection positivity on spheres*, Anal. Math. Phys. 10 (2020), art.no. 9.

[46] K.-H. Neeb, G. Ólafsson: *Algebraic quantum field theory and causal symmetric spaces*, in: *Geometric Methods in Physics XXXIX*, Workshop Białystok 2022, P. Kielanowski et al. (eds.), Trends in Mathematics, Birkhäuser, Cham (2023) 207–231.

[47] K.-H. Neeb, B. Ørsted: *Unitary highest weight representations in Hilbert spaces of holomorphic functions on infinite-dimensional domains*, J. Funct. Anal. 156 (1998) 263–300.

[48] K.-H. Neeb, H. Salmasian: *Differentiable vectors and unitary representations of Frechet-Lie supergroups*, Math. Zeitschrift 275 (2013) 419–451.

[49] K.-H. Neeb, H. Salmasian: *Classification of positive energy representations of the Virasoro group*, Int. Math. Res. Notices 2015/18 (2015) 8620–8656.

[50] K.-H. Neeb, H. Salmasian, C. Zellner: *Smoothing operators and C^*-algebras for infinite dimensional Lie groups*, Internat. J. Math. 28 (2017), art. no. 1750042.

[51] G. I. Olshanskii: *Invariant cones in Lie algebras, Lie semigroups and the holomorphic discrete series*, Funkts. Anal. Prilozhen 15 (1981) 53–66.

[52] G. I. Olshanskii: *Complex Lie semigroups, Hardy spaces and the Gelfand–Gindikin program*, Yaroslav. Gos. Univ., Yaroslavl' (1982) 85–98; English translation: Differential Geom. Appl. 1 (1991) 235–246.

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