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Continuity Characterizing Totally Disconnected Locally Compact Groups

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Abstract. For a locally compact group G and its compact space $\mathcal{SUB}(G)$ of closed subgroups let $\mu_G\colon G \to \mathcal{SUB}(G)$ denote the function which attaches to an element g of G the closed subgroup $\overline{\langle g\rangle}$ generated by it. It is shown that G is totally disconnected if and only if μ_G is continuous. Several other functions which associate with an element of G in a natural way a closed subgroup of G are discussed with respect to their continuity in totally disconnected locally compact groups.
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To each element of the topological group G are naturally associated several closed subgroups of G. This note addresses the continuity of these functions from G to the compact space, $\mathcal{SUB}(G)$, of closed subgroups of G equipped with the Chabauty topology. It is shown that when G is a totally disconnected locally compact group many of these functions are continuous. It is further observed that these functions are generally not continuous when G is not totally disconnected and, indeed, that their continuity sometimes characterises totally disconnected groups among locally compact ones.

The following functions $G \to \mathcal{SUB}(G)$ associate subgroups with g in G:

$$\begin{aligned}
\mu_G(g) &= \overline{\langle g\rangle}, \text{ the closed subgroup generated by } g;\\
\mathrm{lev}_G(g) &= \left\{x \in G \mid \{g^k x g^{-k}\}_{k\in\mathbb{Z}} \text{ is precompact}\right\}, \text{ the \textit{Levi} subgroup of } g; \text{ and}\\
\mathrm{par}_G(g) &= \left\{x \in G \mid \{g^k x g^{-k}\}_{k\in\mathbb{N}} \text{ is precompact}\right\}, \text{ the \textit{parabolic} subgroup of } g.
\end{aligned}$$

It will be shown that each of these functions is continuous for the Chabauty topology on $\mathcal{SUB}(G)$. The proof will use the fact that, in this topology, each closed subgroup H of G has a neighborhood base consisting of sets

$$\{L \in \mathcal{SUB}(G) \mid L \cap K \subseteq WH \text{ and } H \cap K \subseteq WL\}, \tag{1}$$

where K runs over all compact subsets of G and W over all neighborhoods of the identity. Complete details may be found in Bourbaki [2], Chap VIII, §5, n° 6 and

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further information in [1] and [4]. We shall also use without further comment the covariant and contravariant functoriality of $\mathcal{SUB}(-)$ in the following sense.

Proposition. ([5],p. 716 and 717) *Let $\phi\colon G_1 \to G_2$ be a continuous morphism of locally compact groups.*
(i) *If ϕ is proper, then $H \mapsto \phi(H) : \mathcal{SUB}(G_1) \to \mathcal{SUB}(G_2)$ is continuous.*
(ii) *If ϕ is open, then $H \mapsto \phi^{-1}(H) : \mathcal{SUB}(G_2) \to \mathcal{SUB}(G_1)$ is continuous.*

The subgroups $\mathrm{lev}_G(g)$ and $\mathrm{par}_G(g)$ are automatically closed for every g in G, by [9, Proposition 3], and results from [9] together with a development of them made in [3] will be used also in the proofs that lev_G and par_G are continuous functions. In fact, these proofs are straightforward modifications of arguments in [3] showing that the functions $\overline{\mathrm{con}_G}$ and nub_G are continuous functions $G \to \mathcal{SUB}(G)$. The proof of continuity of μ_G will require a different argument.

Theorem 1. *Let G be a totally disconnected, locally compact group. Then each of the functions μ_G, par_G and lev_G is continuous from G to $\mathcal{SUB}(G)$.*

Proof. The proof that par_G is continuous will be given first. Let g be in G, and let $K \subset G$ be compact and let $W \subset G$ be an open neighborhood of the identity. It will be shown that, for every h in that neighborhood, there is a neighborhood of g such that $\mathrm{par}_G(h)$ belongs to the neighborhood of $\mathrm{par}_G(g)$ defined in (1).

By the compactness of K we find a compact open subgroup U so that $xUx^{-1}U \subset W$ for every $x \in K$. By [9, Lemma 1], it may further be supposed, by passing to a subgroup if necessary, that $U = U_+U_-$, that is, that U is tidy above for g. Then [3, Lemma 4.1] shows that, given $u \in U$, there is $t \in U$ such that for every $k \geq 0$ there is $b_k \in U$ such that $t^{-1}(gu)^k t = b_k g^k$.

Consider $x \in \mathrm{par}_G(g) \cap K$. For u and t as in the previous paragraph and $k \geq 0$, there is $b_k \in U$ such that

$$(gu)^k(txt^{-1})(gu)^{-k} = tb_k(g^kxg^{-k})(tb_k)^{-1}.$$

Since tb_k belongs to the compact set U and $\{g^kxg^{-k}\}_{k\geq 0}$ has compact closure, it follows that $\{(gu)^k(txt^{-1})(gu)^{-k}\}_{k\geq 0}$ has compact closure as well, so that txt^{-1} belongs to $\mathrm{par}_G(gu)$. Furthermore, since x belongs to K, we have $x = wtxt^{-1}$ with $w \in W$ and it follows that

$$\mathrm{par}_G(g) \cap K \subset W\mathrm{par}_G(gu).$$

Next consider $y \in \mathrm{par}_G(gu) \cap K$ while u and t are as in the previous paragraph and $k \geq 0$. Then

$$g^k(t^{-1}yt)g^{-k} = b_k^{-1}t^{-1}((gu)^k y(gu)^{-1})tb_k \tag{2}$$

and it follows by arguing as in the other case that

$$\mathrm{par}_G(gu) \cap K \subset W\mathrm{par}_G(g).$$

The set gU is therefore the open neighborhood of g that is claimed to exist in the first paragraph.

The proof that lev_G is continuous relies on [3, Lemma 4.3], which asserts that for each $u \in U \cap g^{-1}Ug$ there is $r \in U$ such that for every $k \in \mathbb{Z}$ there is

$b_k \in U$ such that $r^{-1}(gu)^k r = b_k g^k$. Then a similar argument to that given above shows that

$$\mathrm{lev}_G(g) \cap K \subset W\mathrm{lev}_G(gu) \quad \text{and} \quad \mathrm{lev}_G(gu) \cap K \subset W\mathrm{lev}_G(g)$$

for any $u \in U \cap g^{-1}Ug$.

The proof that μ_G is continuous treats separately the two possibilities for $\overline{\langle g\rangle}$ identified by Weil's Lemma, see [7, Proposition 7.43]. These two possibilities are

Case (i): $\overline{\langle g\rangle}$ is compact.

Case (ii): $\overline{\langle g\rangle}$ is discrete and isomorphic to $\mathbb{Z}$.

The set of all elements of the group G that fall into case (i) and thus generate a compact subgroup of G that is denoted by $\mathrm{comp}(G)$. This set is equal to the set of elements that belong to some compact subgroup of G. It is shown in [10, Theorem 2], that $\mathrm{comp}(G)$ is a closed subset of G and it is clear that $\mathrm{comp}(G)$ is open as well because, as is well known and may be seen in [9, Lemma 7] for example, every compact subgroup of a totally disconnected locally compact group is contained in an open compact subgroup.

Continuity of μ_G on $\mathrm{comp}(G)$ will follow from the next lemma, which establishes a slightly stronger result than needed.

Lemma 2. *If the filter of identity neighborhoods of G has a basis of compact normal subgroups, then μ_G is continuous. In particular, μ_G is continuous if G is profinite.*

Proof. For a given identity neighborhood W we find an open normal subgroup N of G contained in W. Let $g \in G$ and set $U = gN$. Then $h \in U$ implies $hN = gN$ and thus $\overline{\langle h\rangle}N = \overline{\langle g\rangle}N \subseteq W\overline{\langle g\rangle} \cap W\overline{\langle h\rangle}$. Hence $\overline{\langle h\rangle}$ belongs to the neighborhood of $\overline{\langle g\rangle}$ determined as in (1) by W and any compact $K \subseteq G$. ■

Since every compact subgroup of a totally disconnected locally compact group is contained in an open compact subgroup, and since every compact totally disconnected group is profinite, it follows by Lemma 2 that μ_G is continuous at g for every $g \in G$ belonging to $\mathrm{comp}(G)$.

We now turn to the final remaining case, when $g \in G \setminus \mathrm{comp}(G)$ and $\overline{\langle g\rangle}$ is discrete and isomorphic to $\mathbb{Z}$.

Lemma 3. *Suppose that $g \in G$ satisfies $\overline{\langle g\rangle} \cong \mathbb{Z}$. Then μ_G is continuous at g.*

Proof. Let K be an arbitrary compact set and W a given identity neighborhood. Since K is compact and $\overline{\langle g\rangle}$ discrete, $\overline{\langle g\rangle} \cap K$ is finite. Then, since $g \mapsto g^n$ is continuous for all $n \in \mathbb{Z}$, it follows that $\overline{\langle g\rangle} \cap K \subseteq W\overline{\langle h\rangle}$ for all h sufficiently close to g. To complete the proof it must, by (1), also be shown that $\overline{\langle h\rangle} \cap K \subseteq W\overline{\langle g\rangle}$ holds for all h sufficiently close to g. The same argument would apply if it could be established that there are a neighborhood of g and an integer N such that $\overline{\langle h\rangle} \cap K \subseteq h^{[-N,N]}$ for every h in the neighborhood. First replacing K by $K \cup K^{-1}$ if necessary in order to assume that $K = K^{-1}$, this will follow from

(∗) *there is a natural number N and a neighborhood U of g such that for $h \in U$ and all $N < n \in \mathbb{N}$ we have $h^n \notin K$.*

For the proof of $(*)$ we use the second author's structure theory of locally compact totally disconnected groups as exposed in [9]. So we consider a compact open subgroup V which is tidy for g and let $\mathbf{s}\colon G \to \mathbb{N}$ be the scale function of G according to [9], p. 343 and pp. 352ff. If $\mathbf{s}(g) = 1$, then we claim that $gVg^{-1} \subseteq V$. Indeed $V = V_+V_-$, by Theorem 1 in [9] and the definition that follows it, where $gV_-g^{-1} \subseteq V_-$ by definition and $gV_+g^{-1} = V_+$ because $\mathbf{s}(g) = 1$, by Theorem 2 in [9] and the definition that follows it. Now we choose N such that g^n does not belong to the compact set VK when $n > N$. Then, for any $h \in gV$ we have $h^n \subseteq Vg^n$ and so $h^n \notin K$ when $n > N$. Thus $(*)$ is satisfied in this case.

For the case when $\mathbf{s}(g) > 1$, note that $\mathbf{s}(K)$ is bounded because $\mathbf{s}$ is continuous and K is compact. Therefore we find an $N \in \mathbb{N}$ such that $n > \mathbb{N}$ implies $\mathbf{s}(g)^n \notin \mathbf{s}(K)$. As V is tidy for g, for any $h \in gV$ we have $\mathbf{s}(h) = \mathbf{s}(g)$ (see [9], p. 356, Theorem 3). Then, since $\mathbf{s}(h^n) = \mathbf{s}(h)^n$ for all $n \in \mathbb{N}$ (see [9], p. 354, Corollary 3), we have $\mathbf{s}(h^n) \notin \mathbf{s}(K)$ when $n > N$ and so h^n does not belong to K. ■

Lemmas 2 and 3 complete the proof that μ_G is continuous and also the proof of Theorem 1. ■

Remark 1. (a) As noted by the referee, the above proof of the continuity of par_G and lev_G establishes more than stated in Theorem 1. The calculation in (2) shows that $t^{-1}yt$ belongs to $\mathrm{par}_G(g)$ and hence that $\mathrm{par}_G(h)$ is conjugate to $\mathrm{par}_G(g)$ for every h in the neighborhood gU of g. The similar calculation for $\mathrm{lev}_G(gu)$ shows that $\mathrm{lev}_G(h)$ is conjugate to $\mathrm{lev}_G(g)$ for every h in the neighborhood $g(U \cap g^{-1}Ug)$ of g.

(b) It was remarked in the course of the proof of Theorem 1 that the set $\mathrm{comp}(G)$ is open and closed when G is a totally disconnected locally compact group. That is not the case when G is not totally disconnected. For example, $\mathrm{comp}(G)$ is the union of all conjugates of $\mathrm{SO}(2)$ when G is the 3-dimensional connected Lie group $\mathrm{SL}(2,\mathbb{R})$ and is neither open nor closed. Since $\mathrm{lev}_G(g) = \mathrm{par}_G(g) = \mathrm{SL}(2,\mathbb{R})$ if g is in $\mathrm{SO}(2)$ or a conjugate and are proper subgroups otherwise, the same group shows that lev_G and par_G may fail to be continuous when G has a connected component larger than a single point. This example is non-abelian because lev_G and par_G are continuous when G is abelian.

The function μ_G, too, is discontinuous when G is not totally disconnected.

Remark 2. *Let $\mathbb{R}$ denote the additive group of real numbers and $\mathbb{T}$ its quotient group $\mathbb{R}/\mathbb{Z}$. Then*

A. *the function $\mu_\mathbb{R}$ is discontinuous at 0, and*

B. *$\mu_\mathbb{T}$ is discontinuous at all points of $\mathbb{Q}/\mathbb{Z}$.*

Indeed, the sequence $(n^{-1})_{n\in\mathbb{N}}$ in $\mathbb{R}$ converges to 0 but $(\overline{\langle n^{-1}\rangle})_{n\in\mathbb{N}}$ converges to $\mathbb{R} \neq \{0\}$ in $\mathcal{SUB}(\mathbb{R})$. Moreover, $\mu_\mathbb{T}$ is discontinuous on $\mathbb{T}$ in a dramatic way, since it is discontinuous at any element of $\mathbb{Q}/\mathbb{Z} \subseteq \mathbb{R}/\mathbb{Z}$; this follows since $\mu_\mathbb{T}(q + \mathbb{Z})$ is finite cyclic for rational q and equals $\mathbb{T}$ at all other points. Also, it is continuous at every point of $\mathbb{T} \setminus (\mathbb{Q}/\mathbb{Z})$, following from the observation that for a sequence $(a_n/b_n)_{n\in\mathbb{N}}$ of reduced fractions of integers approximating an irrational number has $b_n \to \infty$. (Cf. [1])

Continuity of μ_G in fact characterizes when G is totally disconnected.

Theorem 4. *For a locally compact group G, these conditions are equivalent:*
(1) $\mu_G\colon G\to \mathcal{SUB}(G)$ *is continuous.*
(2) G *is totally disconnected.*

Proof. It has already been shown in Theorem 1 that (2)⇒(1). Remark 1 allows us to prove the reverse implication.

Let G be a locally compact group and assume that μ_G is continuous. The proof will be completed by showing that G_0, the identity component of G, is a single point. The assumption implies that $\mu_{G_0}\colon G_0\to\mathcal{SUB}(G_0)$ is continuous and so it may be assumed for the remainder of the argument that G is connected. If G is not compact, then G contains a closed subgroup isomorphic to $\mathbb{R}$ (see e.g. [6], Theorem 12.81 and recall that all connected locally compact groups are pro-Lie groups by [8], p. 175). Since $\mu_{\mathbb{R}}$ is discontinuous at 0 by Remark 1.A, μ_G is discontinuous in this case. It may therefore be assumed that G is compact and connected. Let T be a maximal closed connected abelian subgroup. If T is singleton, then G is singleton (see [7] Theorem 9.32(ii)). Hence it is no loss of generality for the proof to assume that G is compact, connected, abelian, and nonsingleton. Then G has a surjective continuous character $f\colon G\to\mathbb{T}$. Since μ_G is continuous on G, it follows that $\mu_{\mathbb{T}}$ is continuous on the quotient $\mathbb{T}$.
This is a contradiction to Remark 1.B and thus proves the Proposition. ■

We conclude the note with a couple of remarks about functions to $\mathcal{SUB}(G)$ that are not continuous. First, $\mathrm{lev}_G(g)$ is clearly the intersection of $\mathrm{par}_G(g)$ and $\mathrm{par}_G(g^{-1})$. However, it does not follow that continuity of lev_G can be deduced from that of par_G and the inverse map because the map

$$\mathcal{SUB}(G)\times\mathcal{SUB}(G)\to\mathcal{SUB}(G) : (K,L)\mapsto K\cap L$$

is not continuous. Let $G=\mathbb{Z}_p\times\mathbb{Z}_p$, where $\mathbb{Z}_p$ is the additive group of p-adic integers, and consider the closed subgroups $A_n=\mathbb{Z}_p\times\{0\}$ and $B_n=\mathbb{Z}_p.(1,p^n)$ for $n=0,1,2,\dots$. Then

$$\lim_{n\to\infty}A_n=\lim_{n\to\infty}B_n=\mathbb{Z}_p\times\{0\}=(\lim_{n\to\infty}A_n)\cap(\lim_{n\to\infty}B_n)$$

but $A_n\cap B_n=\{(0,0)\}$ for all n and so $\lim_{n\to\infty}(A_n\cap B_n)=\{(0,0)\}$.

Finally, lest it be thought that naturally defined functions $G\to\mathcal{SUB}(G)$ are always continuous when G is totally disconnected, we remark that C_G, the centralizer map is rarely continuous at the identity if G is not discrete. The centralizer of the identity element is always G but if, for example, $G=\mathrm{SL}(2,\mathbb{Z}_p)$, it is not the case that $C_G(g)\to G$ as $g\to 1_G$.

The centralizer function $C_G : G\to\mathcal{SUB}(G)$ is continuous if G has a central open subgroup but that is not the most general class of groups for which that is the case. Recall that the *quasi-center* of a topological group G is the set of elements that centralize an open subgroup of G. The quasi-center is a subgroup of G but might not be closed. For example, $F^{\mathbb{N}}$, which F a finite group with trivial center, has dense quasi-center but many elements of $F^{\mathbb{N}}$ do not centralize an open subgroup.

Proposition 5. *Let G be a topological group and suppose that the quasi-center of G is dense. Then the centralizer function $C_G : G \to \mathcal{SUB}(G)$ is continuous at all points in the quasi-center of G.*

Proof. Let $g \in G$ and let K and W be respectively a compact subset and a neighborhood of the identity in G.

Choose an open subgroup $U \subseteq W$. Then K is covered by a finite number of U-cosets. Indeed, replacing K by UK, it may be supposed that K is the union of a finite set of U-cosets. Let $\{V_\lambda\}_{\lambda\in\Lambda}$ be a filter of compact open subgroups that converges to the identity. For each $\lambda \in \Lambda$ put

$$S_\lambda = \{Ux \subset K \mid \exists h_\lambda \in gV_\lambda \text{ with } C_G(h_\lambda) \cap Ux \neq \emptyset\}.$$

Then S_λ is a descending family of finite sets and so there is $\lambda_0 \in \Lambda$ such that $S_\lambda = S_{\lambda_0}$ for every $\lambda \geq \lambda_0$. Consider $Ux \in S_{\lambda_0}$. For every $\lambda \geq \lambda_0$ there is $y_\lambda \in h_\lambda \in gV_\lambda$ and $C_G(h_\lambda) \cap Ux \neq \emptyset$. Let y be a limit point of $\{y_\lambda\}_{\lambda\in\Lambda}$ and assume, without loss, that in fact $y_\lambda \to y$. Then $y \in Ux$ and $gyg^{-1}y^{-1} = \lim_\lambda h_\lambda y_\lambda h_\lambda^{-1} y_\lambda^{-1} = 1$. Hence $C_G(g) \cap Ux \neq \emptyset$ for every $Ux \in S_{\lambda_0}$ and it follows that $S_{\lambda_0} \subseteq UC_G(g)$. By the choice of λ_0, if $h \in gV_{\lambda_0}$, then $C_G(h) \cap K \subseteq S_{\lambda_0}$ and it has therefore been shown that $C_G(h) \cap K \subseteq WC_G(g)$ for every $h \in gV_{\lambda_0}$. (Note that this argument does not use the hypothesis that g is in the quasi-center of G and so applies to any element.)

For the other inclusion needed in (1), assume that g belongs to the quasi-center of G and let U be a compact open subgroup of G that is contained in $W \cap C_G(g)$. Then $C_G(g) \cap UK$ is a union of U-cosets and is open. Since the quasi-center of G is dense, there is a finite set $\mathcal{K} \subset UK$ in the quasi-center such that $C_G(g) \cap UK = U\mathcal{K}$. Let V be a compact open subgroup that is centralized by $\mathcal{K}$. Then $\mathcal{K} \subseteq C_G(h)$ for every h in gV. Hence

$$C_G(g) \cap K = U\mathcal{K} \subset UC_G(h) \subset WC_G(h)$$

for every $h \in gV$ and it follows that C_G is continuous at g. ∎

The (topological) normal closure map, $\mathrm{ncl}_G \colon G \to \mathcal{SUB}(G)$, $\mathrm{ncl}_G(g) = \overline{\langle xgx^{-1} \mid x \in G\rangle}$ also fails to be continuous at the identity in general. The normal closure of the identity is the trivial subgroup but if, for example, G is topologically simple, then $\mathrm{ncl}_G(g) = G$ whenever $g \neq 1_G$. Here is a complement to Theorem 4 for normal closures.

Proposition 6. *Let G be a locally compact group that has a neighborhood base at the identity of compact open normal subgroups. Then $\mathrm{ncl}_G \colon G \to \mathcal{SUB}(G)$ is continuous.*

Proof. Let $g \in G$. Then for any compact open normal subgroup, V, of G and any $h \in gV$, we have that $\mathrm{ncl}_G(h) \subseteq V\mathrm{ncl}_G(g)$ and $\mathrm{ncl}_G(g) \subseteq V\mathrm{ncl}_G(h)$. Since G has a base at the identity of compact open normal subgroups, it follows that $\mathrm{ncl}_G \colon G \to \mathcal{SUB}(G)$ is continuous at g. ∎

The converse to Proposition 5 fails. For example, let D_∞ be the infinite dihedral group $\mathbb{Z} \rtimes \{1, -1\}$, let C be an order two subgroup and set

$$G = \left\{g \in D_\infty^{\mathbb{N}} \mid g(n) \in C \text{ for all but finitely many } n\right\}$$

equipped with the topology such that $C^{\mathbb{N}}$ is a compact open subgroup. Then G has no compact open normal subgroups but ncl_G is continuous.

The problem of a precise determination for which groups such functions as the centralizer function C_G or the normal closure function ncl_G are continuous remains open at this time.

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