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Erratum of "Isomorphy Classes of Involutions of $\mathrm{SP}(2n,k)$, $n>2$"

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Abstract. In *Isomorphy Classes of Involutions of* $\mathrm{SP}(2n,k)$, $n>2$, which was published in Volume 25 of the Journal of Lie Theory, we developed a detailed characterization of the involutions of $\mathrm{SP}(2n,k)$. We used these results to classify the isomorphy classes of involutions of $\mathrm{SP}(2n,k)$ where k is any field not of characteristic 2. It has become apparent that there are two similar errors in this paper that occur in some technical lemmas. We correct these errors here. These corrections preserve the results of the paper.
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1. Type 3 Involutions

Lemma's 4.10 and 4.11 should both assume $i \notin k$. Because of this, the proof of Lemma 4.12 cannot rely upon Lemma 4.11. So, we must construct bases for $E(A,i)$ and $E(A,-i)$ in k^{2n} in the proof of Lemma 4.12. This can be done in a similar fashion as to what is done in the proof of Lemma 4.11. Specifically, the first sentence of the proof of Lemma 4.12 should be replaced with the following:

Proof. We are now assuming $i \in k$. Since Inn_A is Type 3, then we are assuming that $A \in \mathrm{SP}(2n,k)$ and $A^2 = -I$. It follows that all eigenvalues of A are $\pm i$. Since there are no repeated roots in the minimal polynomial of A, then we see that A is diagonalizable. We wish to construct bases for $E(A,i)$ and $E(A,-i)$ such that all the vectors lie in k^{2n}. Let $\{z_1,...,z_{2n}\}$ be a basis for k^{2n}. For each j, let $u_j = (A+iI)z_j$. Note that

$$Au_j = A(A+iI)z_j = (A^2+iA)z_j = (-I+iA)z_j = i(A+iI)z_j = iu_j.$$

So, $\{u_1,...,u_{2n}\}$ must span $E(A,i)$. Thus, we can appropriately choose $m = \dim(E(A,i))$ of these vectors and form a basis for $E(A,i)$. We can renumber, and assume that the m chosen vectors are $u_1,...,u_m$ are such that we can write $u_j = (A+iI)z_j$.

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Let $u_j' = (A - iI)z_j$ for $j = 1, ..., m$. We can show that these vectors are linearly independent, and lie in $E(A, i)$. Note that this shows $\dim(E(A, i)) \geq \dim(E(A, -i))$. We can similarly show the reverse inequality, so we have that $\dim(E(A, i)) = \dim(E(A, -i))$, and $m = n$. Further, the u_j''s form a basis for $E(A, i)$ in k^{2n}. ■

2. Type 4 Involutions

The error in this section is analogous to the error in the previous section, but the correction is slightly more complicated. Firstly, Lemmas 4.17 and 4.18 should be replaced by the following Lemma:

Lemma 2.1. *Suppose* $\theta = \mathrm{Inn}_A$ *is a Type 4* k*-involution of* $\mathrm{SP}(2n, k)$ *where* $A \in \mathrm{SP}(2n, k[\sqrt{\alpha}])$. *Then, we can find* $x_1, ..., x_{\frac{n}{2}}, y_1, ..., y_{\frac{n}{2}} \in k^{2n}$ *such that the* $x_j + \sqrt{-\alpha} y_j$ *are a basis for* $E(A, -i)$ *and the* $x_j - \sqrt{-\alpha} y_j$ *are a basis for* $E(A, i)$. *Further,* $\dim(E(A, i)) = \dim(E(A, -i))$.

If $\sqrt{-\alpha} \in k$, *then these bases lie in* k^{2n}. *Otherwise, they lie in* $k[\sqrt{-\alpha}]^{2n}$.

Proof. Since Inn_A is Type 4, then we are assuming that $A^2 = -I$. It follows that all eigenvalues of A are $\pm i$. Since there are no repeated roots in the minimal polynomial of A, then we see that A is diagonalizable. We begin by constructing bases for $E(A, i)$ and $E(A, -i)$ such that all the basis vectors lie in $k[\sqrt{-\alpha}]^{2n}$. Let $\{z_1, ..., z_{2n}\}$ be a basis for k^{2n}. For each j, let $u_j = (\sqrt{\alpha} A - \sqrt{-\alpha} I) z_j$. Note that

$$\begin{aligned} Au_j &= A(\sqrt{\alpha} A - \sqrt{-\alpha} I) z_j \\ &= (\sqrt{\alpha} A^2 - \sqrt{-\alpha} A) z_j \\ &= -i(\sqrt{\alpha} A - \sqrt{-\alpha} I) z_j \\ &= -iu_j. \end{aligned}$$

So, $\{u_1, ..., u_{2n}\}$ must span $E(A, -i)$. Thus, we can appropriately choose $m = \dim(E(A, -i))$ of these vectors and form a basis for $E(A, -i)$. Note that each of these vectors lies in $k[\sqrt{-\alpha}]^{2n}$. We can renumber, and assume that the m chosen vectors $u_1, ..., u_m$, are such that we can write $u_j = (A + iI)z_j$. We can similarly show that $u_j' = (A - iI)z_j \in E(A, i)$, and that the set $u_1', ..., u_m'$ is linearly independent. So, $\dim(E(A, i)) \geq \dim(E(A, -i))$. We can similarly show the reverse inequality. It follows from these observations that $\dim(E(A, i)) = n = \dim(E(A, -i))$, and that the v_j''s form a basis for $E(A, i)$ in $k[\sqrt{-\alpha}]^{2n}$. If $\sqrt{-\alpha} \in k$, then these bases lie in k^{2n}. We have shown what was needed. ■

Any proof that cites Lemma 4.17 or Lemma 4.18 should instead cite this lemma.

References

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