Journal of Lie Theory
Volume **26** (2016) 1199–1200

Erratum to "Isomorphism Classes of k-Involutions of Algebraic Groups of Type F_4"

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Communicated by G. Ólafsson

Abstract. In Journal of Lie Theory **25**, Issue 4, the paper entitled *Isomorphism classes of k-involutions of algebraic groups of type* F_4 appeared with an error in the statement of Lemma 4.5, and the subsequent proof is invalid. The results that follow in the original paper, however, are implied by the statements of this note, which therefore effectively replace Lemma 4.5 of the original paper.
Mathematics Subject Classification 2010: 20G15, 20G41, 17C30.
Key Words and Phrases: Exceptional groups, Jordan algebras, generalized symmetric spaces, algebraic groups.

1. Certain involutions in $\mathrm{Aut}(\mathcal{A})$

In *Isomorphism classes of k-involutions of algebraic groups of type* F_4 Lemma 4.5 [1] is stated as an *if and only if* statement that isn't true without making some assumptions on the k-algebra $\mathcal{A}$ and theautomorphism $t \in \mathrm{Aut}(\mathcal{A})$. In particular the characteristic of k can not be 2, and we now assume there is a decomposition of $\mathcal{A}$ with respect to a nondegenerate bilinear form that is left invariant by our automorphism of order 2. The following three statements are meant to replace Lemma 4.5 and the remainder of the results in *Isomorphism classes of k-involutions of algebraic groups of type* F_4 follow from these statements. The following statements are also included in [2].

Lemma 1.1. *Let $\mathcal{A}$ be an algebra. If $t, t' \in \mathrm{Aut}(\mathcal{A})$ are $\mathrm{Aut}(\mathcal{A})$-conjugate, then their respective fixed point subalgebras $\mathcal{D}, \mathcal{D}' \subset \mathcal{A}$ are isomorphic.*

Proof. We start by assuming there exists a $g \in \mathrm{Aut}(\mathcal{A})$ such that $gt = t'g$. Take $a \in \mathcal{D}$, the fixed point subalgebra of $\mathcal{A}$ with respect to t,

$$gt(a) = t'g(a)\,, \quad g(a) = t'g(a)\,,$$

and $g(a) \in \mathcal{D}'$, the fixed point subalgebra with respect to t'. Now we just reverse

ISSN 0949–5932 / \$2.50

the argument using the fact that $g \in \mathrm{Aut}(\mathcal{A})$ and $tg^{-1} = g^{-1}t'$. This shows us that $g^{-1}(a') \in \mathcal{D}'$ for all $a' \in \mathcal{D}'$, and so $g(\mathcal{D}) = \mathcal{D}'$. ∎

Before we look at the partial converse we consider the following situation.

Proposition 1.2. *Let $\mathcal{A}$ is a k-algebra where k is a field and $\mathrm{char}(k) \neq 2$. If $t \in \mathrm{Aut}(\mathcal{A})$, $t^2 = \mathrm{id}$, D is the subalgebra of $\mathcal{A}$ fixed by t, and*

$$\mathcal{A} = \mathcal{D} \oplus \mathcal{D}^{\perp},$$

with respect to a nondegenerate bilinear form that is left invariant by t, then for $b \in D^{\perp}$ we have $t(b) = -b$.

Proof. Let $\mathcal{A} = \mathcal{D} \oplus \mathcal{D}^{\perp}$ such that $\mathcal{D}$ is the fixed point subalgebra of $t \in \mathrm{Aut}(\mathcal{A})$ such that $t^2 = \mathrm{id}$. Let $b \in D^{\perp}$; then $t(b+t(b)) = t(b)+b = b+t(b) \Rightarrow b+t(b) \in D$. Now if we choose $a \in \mathcal{D}$ we see that $\langle a, b+t(b)\rangle = \langle a, b\rangle + \langle t(a), t(b)\rangle = 0$, since t is an isometry, so $b+t(b) \in \mathcal{D}^{\perp}$. We have $b+t(b) \in \mathcal{D} \cap \mathcal{D}^{\perp} = \{0\}$, so $t(b) = -b$. ∎

Lemma 1.3. *Let $\mathcal{A}$ be a k-algebra with a nonsingular bilinear form where k is a field not of characteristic 2. Assume $t, t' \in \mathrm{Aut}(\mathcal{A})$ are elements of order 2 that have fixed point subalgebras $\mathcal{D}, \mathcal{D}'$ respectively, and $g(\mathcal{D}) = \mathcal{D}'$ for $g \in \mathrm{Aut}(\mathcal{A})$. If g leaves the bilinear form on $\mathcal{A}$ invariant and*

$$\mathcal{A} = \mathcal{D} \oplus \mathcal{D}^{\perp} = \mathcal{D}' \oplus \mathcal{D}'^{\perp}.$$

Then t and t' are $\mathrm{Aut}(\mathcal{A})$-conjugate.

Proof. Let $\mathcal{D}, \mathcal{D}' \subset \mathcal{A}$ such that $\mathcal{D}$ and $\mathcal{D}'$ are the fixed point subalgebras of $t, t' \in \mathrm{Aut}(\mathcal{A})$ respectively, and $t^2 = t'^2 = \mathrm{id}$. Let $g \in \mathrm{Aut}(\mathcal{A})$ be such that $g(\mathcal{D}) = \mathcal{D}'$. Suppose $a \in \mathcal{D}$ and $b \in \mathcal{D}^{\perp}$. We assume that $\mathcal{A}$ has the following decomposition, $\mathcal{A} = \mathcal{D} \oplus \mathcal{D}^{\perp} = \mathcal{D}' \oplus \mathcal{D}'^{\perp}$. Notice $g(\mathcal{D}'^{\perp}) = \mathcal{D}^{\perp}$, and we take $a \in \mathcal{D}$ and $b \in \mathcal{D}^{\perp}$, then by 1.2 we have

$$t'g(a+b) = g(a) - g(b) = g(a-b) = gt(a+b).$$ ∎

References

[1] Hutchens, J., *Isomorphism Classes of k-Involutions of Algebraic Groups of Type F_4*, J. of Lie Theory 25 (2015), 1003–1022.

[2] —, *Isomorphism classes of k-involutions of algebraic groups of type E_6*, Beiträge zur Algebra und Geometrie 2016, to appear.

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Received January 16, 2016
and in final form May 27, 2016