

On the Cuspidal Support of a Generic Representation

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Abstract. We prove that, for p -adic quasi-split classical groups, generic representations in an L -packet have the smallest cuspidal support among all the representations in that L -packet.

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1. Introduction

Since the seminal work of Arthur [1] (and later of Mok ([10])), there is an increased effort to understand representations in local Arthur packets (in many directions; e.g. to relate Mœglin's parametrization with the Arthur's work, mostly done by Mœglin herself). The general characterization of the representations in Arthur's class (i.e., those irreducible admissible representations which occur as local components of automorphic representations) is still unknown.

Along this lines lies the study of Langlands parameters and L -packets. Although better understood than A-packets, there are many questions still unresolved, e.g., understanding representation from different Bernstein components inside one L -packet. This question (and a generalization of it on the Galois side of the Langlands parameter) was addressed in [3].

In this short note we prove, roughly, that in a generic L -packet for a classical quasi-split p -adic group, the generic representations have the smallest cuspidal support. This is, somehow, the simplest case where one can characterize the relation between different Bernstein components of the representations in one L -packet. This result is certainly expected by experts, but we were not able to find a reference for it, and we hope that writing down the proof of this fact will be helpful in deeper understanding of the structure of L -packets.

To prove this result, we mainly use a claim we proved in Proposition 3.1 of [6] (and Remark after it, cf. also the sixth section of [6]). This claim follows directly from Mœglin-Tadić classification of the discrete series for classical groups

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and can be briefly explained as follows: the ϵ factor attached to a generic discrete series equals one. This ϵ factor is one of the invariants of the discrete series in Mœglin-Tadić classification and it can be related to (we do not need this in this paper) the character of the component group of the centralizer of the Langlands parameter (this character is one the ingredients of the Langlands parametrization of all the irreducible representations of p -adic reductive groups). Thus, essentially, the claim in [6] amounts to the claim that generic representations in an L -packet correspond to the trivial character (of the component group), which is now reproved by Atobe ([2]), directly from Arthur's classification.

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2. Notation

Let F be a non-archimedean local field of characteristic zero. Let G_n denote a group in one of the following series of classical, quasi-split groups (here n denotes the semisimple F -rank of that group): full even orthogonal groups if that group is split; otherwise it denotes special even orthogonal groups, odd special orthogonal groups, symplectic groups, unitary groups. These series of groups are precisely the (quasi-split) ones whose discrete series representations were classified in [9] and which were discussed in [5]. We will work exclusively with admissible, finite length (complex) representations of the groups G_n .

Let $P = MN$ be a parabolic subgroup of G_n with a Levi factor M and the unipotent radical N . Then $M \cong GL(n_1, F) \times GL(n_2, F) \times \cdots \times GL(n_k, F) \times G_{n'}$, where $n_1, \dots, n_k, n' \in \mathbb{N}_0$ are such that $n_1 + n_2 + \cdots + n_k + n' = n$. Then, if π_i , $i = 1, \dots, k$ is an admissible representation of $GL(n_i, F)$ and σ' is an admissible representation of $G_{n'}$, we denote, following Zelevinsky and Tadić, a parabolically induced representation (the normalized induction) $\text{Ind}_P^{G_n}(\pi_1 \otimes \cdots \otimes \pi_k \otimes \sigma')$ by $\pi_1 \times \cdots \times \pi_k \rtimes \sigma'$. For any $n \geq 1$, we denote by ν a character of $GL(n, F)$ given as a composition of the determinant character of $GL(n, F)$ with the modulus character of F^* . Let ρ be an irreducible cuspidal representation of some $GL(m, F)$. Then, the induced representation $\rho\nu^{k-1} \times \rho\nu^{k-2} \times \cdots \times \rho$ of $GL(mk, F)$ has a unique irreducible subrepresentation, which we denote by $\delta[\rho, \rho\nu^{k-1}]$. This representation is essentially square-integrable.

For more information on the notion of the cuspidal support of an irreducible admissible representation σ of G_n we refer to [4]. We now just briefly recall the definition. For each irreducible admissible representation σ of G_n , there exists a standard parabolic subgroup $P = MN$ and an irreducible cuspidal representation $\pi := \pi_1 \otimes \cdots \otimes \pi_k \otimes \sigma'$ of M (with notation as in preceding paragraph) such that σ is a subquotient of $\pi_1 \times \cdots \times \pi_k \rtimes \sigma'$. A subgroup M and a representation π with this property are unique, up to associativity. This associativity class of (M, π) is called the cuspidal support of σ . Thus, we have an associativity class of standard parabolic subgroups on which the cuspidal support of σ is supported (note that we consider full split even orthogonal groups).

We use results of Jantzen ([7]) on discrete series of classical groups and their cuspidal supports: let ρ_1, \dots, ρ_k denote irreducible unitary cuspidal representations of $GL(n_i, F)$, $i = 1, \dots, k$, with $\rho_i \not\cong \rho_j, \tilde{\rho}_j$, for $i \neq j$. Let $S(\rho_i) = \{\rho_i \nu^\alpha, \tilde{\rho}_i \nu^\alpha : \alpha \in \mathbb{R}\}$. If a discrete series σ of G_n has a cuspidal support in $\bigcup_{i=1}^k S(\rho_i) \cup \sigma_{\text{cusp}}$, where σ_{cusp} is an irreducible cuspidal representation of $G_{n'}$, $n' \leq n$, then, for every $j = 1, \dots, k$, there exists an irreducible representation σ_j (completely determined by σ) and a representation π_j such that $\sigma \hookrightarrow \pi_j \rtimes \sigma_j$, where the cuspidal support of σ_j consists only of representations of $S(\rho_j)$ (and σ_{cusp}) and π_j does not have elements of $S(\rho_j)$ in the cuspidal support. Jantzen proved that σ is square-integrable if and only if σ_j is square-integrable, for all $j = 1, \dots, k$. For a self-dual unitary cuspidal representation ρ of $GL(n_\rho, F)$ and an irreducible square-integrable representation σ of G_n , we denote by σ_ρ a representation whose cuspidal support is in $S(\rho)$ (and σ_{cusp}) and which we just described. Because of that, we will often write "the representation σ is (ρ) -strongly positive" meaning that the component σ_ρ is strongly positive (or sometimes, in this situation, we will just say "strongly positive", without mentioning ρ). It will also be clear from the context when cuspidality of σ actually means the cuspidality of σ_ρ .

We fix a non-degenerate character χ of the unipotent radical of a Borel subgroup of G_n defined over F . We fix such a choice of a Borel subgroup; this gives us standard parabolic subgroups. When we say "generic representation" it means χ -generic representation.

By the work of Arthur it follows that the Jordan blocks introduced by Mœglin and used in [9] to describe discrete series representations of classical groups G_n are precisely the L -parameters of these discrete series. This is proved in [8], Theorem 1.3.1. For additional explanations regarding the unitary group case, one can check the seventh and the eighth section of [5].

So, in other words, in this note we prove that, among all the discrete series of a group G_n having the same Jordan block, the generic one has the smallest cuspidal support. More precisely, up to the associativity, the parabolic subgroup on which the cuspidal support lives, is the smallest for the generic representations in one L -packet.

Beside this fact about Jordan blocks, to prove our claim, we only use some basic facts about L -parameters (we even do not need some detail knowledge about it) and the following result. ϵ factor is one of the ingredients in the admissible triples used by Mœglin and Mœglin-Tadić ([9]) to describe discrete series representations. Then, by Proposition 3.1 of [6] and Remark after it, the ϵ factor (when defined) on each member of the Jordan block of a generic discrete series attains value 1 (cf. a slight modification of this claim in the case of the split full even orthogonal group is explained in the proof of Proposition 6.7 of [6]); it always attains value 1 on pairs of appropriate elements in the Jordan block of a generic discrete series. We do not need here any further details about this ϵ -factor; we only need the fact that if it assumes value one on the subsequent elements in the Jordan block of a discrete series, then this discrete series can be embedded in a certain induced representation (cf. [9], Lemma 5.1). For a concise overview of construction of the discrete series given in [9] we refer to Preliminaries section of

[11].

From now on, let $\phi : WD_F \rightarrow \hat{G}_n$ be a discrete L -parameter of a group G_n (except in the case of unitary G_n); let Π_ϕ be the corresponding L -packet. Here WD_F denotes the Weil-Deligne group $W_F \times SL(2, \mathbb{C})$ of F , where W_F is the Weil group of F . In the case of unitary groups we just take, instead of \hat{G}_n , the group $GL(N, \mathbb{C})$ for certain N , as explained in Proposition 7.3. of [5], also cf. Theorem 8.1. and the tenth section of [5]. We have

$$\phi = \bigoplus_{(\rho, a) \in \text{Jord}(\sigma)} \rho \otimes V_a,$$

for some $\sigma \in \Pi_\phi$. Here ρ denotes an irreducible representation of W_F (satisfying some properties we do not recall; we call it "admissible") and V_a denotes the unique algebraic representation of $SL(2, \mathbb{C})$ of dimension a . By the Langlands correspondence for the general linear groups, we can identify an irreducible admissible m -dimensional representation ρ of W_F with an irreducible unitary cuspidal representation of $GL(m, F)$. This identification between the L -parameter ϕ and $\text{Jord}(\sigma)$ is the content of Theorem 1.3.1. of [8].

3. The claim

Lemma 3.1. *Let σ_0 be a cuspidal generic representation belonging to an L -packet ϕ of G_n . Then, all the other members of that packet are also cuspidal.*

Proof. The form of the Jordan block (i.e., of the L -parameter) of a cuspidal representation is described in the Introduction of [9]. Since σ_0 is generic, the reducibility of the representation $\rho\nu^\alpha \rtimes \sigma_0$ (where $\rho \cong \tilde{\rho}$ is an irreducible cuspidal representation of $GL(n_\rho, F)$ and $\alpha \in \mathbb{R}$) is generic, i.e., we can have reducibility only if $\alpha \in \{0, \pm\frac{1}{2}, \pm 1\}$. This means that only reducibilities at $\alpha = \pm 1$ contribute to the L -parameter, i.e.

$$\phi \leftrightarrow \text{Jord}(\sigma_0) = \rho_1 \otimes V_1 \oplus \rho_2 \otimes V_1 \oplus \cdots \oplus \rho_k \otimes V_1. \quad (1)$$

Here ρ_i , $i = 1, \dots, k$ is a self-dual, cuspidal representation of $GL(n_i, F)$ (this defines n_i). Note that in order for ϕ to be a discrete parameter, all ρ_i have to be of the same type (orthogonal or symplectic) and, if $i \neq j$ then $\rho_i \not\cong \rho_j$. Let $\sigma \in \Pi_\phi$. The inductive procedure of forming discrete series by starting from the strongly positive ones and then adding two by two elements to the Jordan block is concisely described in Preliminaries section of [11]. From (1) we see that $\text{Jord}_\rho(\sigma)$ is at most singleton for any self-dual cuspidal ρ . From the general construction of discrete series we have just mentioned it follows that σ is itself necessarily strongly positive. We use the notation Jord_ρ also used in [11]. Since Jord_{ρ_i} , $i = 1, \dots, k$ consists of odd numbers (i.e. all 1's), by Theorem 1.1 (ii) of [11], and the discussion preceding it, it follows that σ is necessarily cuspidal. ■

Lemma 3.2. *Let σ_0 be a non-cuspidal, generic, strongly positive discrete series representation belonging to an L -packet Π_ϕ of G_n . Let P_0 be a standard parabolic subgroup attached to the cuspidal support of σ_0 . If $\sigma \in \Pi_\phi$ has its cuspidal support attached to a standard parabolic P , then, up to the associativity, $P_0 \subset P$.*

Proof. Assume that for some self-dual cuspidal GL-representation ρ we have $|\text{Jord}_\rho(\sigma_0)| \geq 2$. Then, on one hand, since σ_0 is strongly positive, the value of ϵ -factor on each pair of subsequent elements in $\text{Jord}_\rho(\sigma_0)$ is equal to -1 (cf. the definition of the triple of alternated type in before Remark 1.1 of [11]), and on the other hand, since σ_0 is generic, by Proposition 3.1 of [6], it has to be 1. Thus, if $\text{Jord}_\rho(\sigma_0) \neq \emptyset$, then $\text{Jord}_\rho(\sigma_0)$ is a singleton. Thus we have

$$\phi \leftrightarrow \text{Jord}(\sigma_0) = \rho_1 \otimes V_{a_1} \oplus \rho_2 \otimes V_{a_2} \oplus \cdots \oplus \rho_k \otimes V_{a_k}.$$

We denote the partial cuspidal support of σ_0 by σ_{cusp} . This is also a generic representation.

As mentioned above, the ϵ factor of a discrete series representation σ is defined on some subset of $\text{Jord}(\sigma) \cup \text{Jord}(\sigma) \times \text{Jord}(\sigma)$. Since we are dealing here with several representations belonging to the same L -packet simultaneously, we denote the ϵ -factor of σ acting on (ρ, a) by $\epsilon_{\sigma, \rho}(a)$.

If a_i is even, then $\epsilon_{\sigma_0, \rho_i}(a_i) = 1$ so that

$$(\sigma_0)_{\rho_i} \hookrightarrow \delta[\rho_i \nu^{1/2}, \rho_i \nu^{\frac{a_i-1}{2}}] \rtimes \sigma_{\text{cusp}}, \quad (2)$$

and if a_i is odd, then $|\text{Jord}_{\rho_i}(\sigma_0)| = |\text{Jord}_{\rho_i}(\sigma_{\text{cusp}})| = 1$, so that $\text{Jord}_{\rho_i}(\sigma_{\text{cusp}}) = \{1\}$. In that case,

$$(\sigma_0)_{\rho_i} \hookrightarrow \delta[\rho_i \nu^1, \rho_i \nu^{\frac{a_i-1}{2}}] \rtimes \sigma_{\text{cusp}}. \quad (3)$$

Let $\sigma \in \Pi_\phi$ be another discrete series with the partial cuspidal support σ'_{cusp} . Then, if a_i is even with $\epsilon_{\sigma, \rho_i}(a_i) = 1$, then $\text{Jord}_{\phi_i}(\sigma'_{\text{cusp}}) = \emptyset$ (cf. before Remark 1.1 in [11]) and we again have

$$(\sigma)_{\rho_i} \hookrightarrow \delta[\rho_i \nu^{1/2}, \rho_i \nu^{\frac{a_i-1}{2}}] \rtimes \sigma'_{\text{cusp}}.$$

If, for even a_i , we have $\epsilon_{\sigma, \rho_i}(a_i) = -1$, then $\text{Jord}_{\phi_i}(\sigma'_{\text{cusp}}) = \{2\}$, so that

$$(\sigma)_{\rho_i} \hookrightarrow \delta[\rho_i \nu^{3/2}, \rho_i \nu^{\frac{a_i-1}{2}}] \rtimes \sigma'_{\text{cusp}}. \quad (4)$$

If a_i is odd, we have $\text{Jord}_{\phi_i}(\sigma'_{\text{cusp}}) = \{1\}$, so that

$$(\sigma)_{\rho_i} \hookrightarrow \delta[\rho_i \nu^1, \rho_i \nu^{\frac{a_i-1}{2}}] \rtimes \sigma'_{\text{cusp}}.$$

We denote by $GL(n, F)^f$ the product of f copies of $GL(n, F)$. The above discussion (with Theorem 1.1 (ii) of [11]) gives us that the (standard) Levi subgroup attached to the cuspidal support of σ_0 is (up to the associativity) given by

$$\prod_{a_i \text{ even}} GL(n_{\rho_i}, F)^{\frac{a_i}{2}} \times \prod_{a_i \text{ odd}} GL(n_{\rho_i}, F)^{\frac{a_i-1}{2}} \times G_d.$$

Here σ_{cusp} is a representation of G_d . We see that the Levi subgroups attached to the cuspidal support of σ can differ from the one for σ_0 only in a situation (4). If this case occurs for some i , that means that in the GL -part of the cuspidal support of σ there is one less copy of $GL(n_{\rho_i}, F)$, for each such i , but then the classical part of the Levi is $G_{d'}$, where now $d' - d$ equals the sum of n'_{ρ_i} 's for which this situation occurs, and this proves our claim. \blacksquare

Now we resolve the general case of the generic discrete series by induction over number of additions of two elements of in the Jordan block by which this discrete series is formed. As any discrete series of the groups G_n is formed starting from a strongly positive discrete series of some smaller rank group $G_{n'}$ by adding two more elements in the Jordan block ([9]), we have already proved our basic induction step.

Proposition 3.3. *Let σ_0 be a generic discrete series representation belonging to an L -packet Π_ϕ of G_n . Let P_0 be a standard parabolic subgroup attached to the cuspidal support of σ_0 . If $\sigma \in \Pi_\phi$ has its cuspidal support attached to a standard parabolic P , then, up to the associativity, $P_0 \subset P$.*

Proof. We can assume that σ_0 is not strongly positive, so that there exists ρ such that $|\text{Jord}_\rho(\sigma_0)| \geq 2$. Let $\sigma \in \Pi_\phi$. First, assume that there exists some ρ' such that $|\text{Jord}_{\rho'}(\sigma)| \geq 2$ with the property that there exist two consequent numbers in $\text{Jord}_{\rho'}(\sigma)$, say a and b , with $a < b$ satisfying $\epsilon_{\sigma, \rho'}(a)\epsilon_{\sigma, \rho'}(b) = 1$. This means that there exists another discrete series, say σ' , with $\text{Jord}(\sigma') = \text{Jord}(\sigma) \setminus \{(\rho', a), (\rho', b)\}$ such that (cf. [9], Lemma 5.1)

$$\sigma \hookrightarrow \delta[\rho'\nu^{-\frac{a-1}{2}}, \rho'\nu^{\frac{b-1}{2}}] \rtimes \sigma'.$$

On the other hand, since σ_0 is generic we necessarily have $\epsilon_{\sigma_0, \rho'}(a)\epsilon_{\sigma_0, \rho'}(b) = 1$ and then there exists a generic representation σ'_0 which belongs to the same L -packet as σ' (\iff the same Jordan block) such that

$$\sigma_0 \hookrightarrow \delta[\rho'\nu^{-\frac{a-1}{2}}, \rho'\nu^{\frac{b-1}{2}}] \rtimes \sigma'_0.$$

Now the claim of the proposition directly follows from the analogous claim for σ'_0 and σ' , which was our inductive assumption.

If there is no such ρ' (with such a and b), it means that σ is strongly positive. We denote the partial cuspidal support of σ by σ_{cusp} . There exists a bijection between $\text{Jord}_\rho(\sigma)$ and $\text{Jord}_\rho(\sigma_{\text{cusp}})$ or $\text{Jord}_\rho(\sigma_{\text{cusp}}) \cup \{0\}$, as explained before Remark 1.1 of [11]. We denote this bijection by ϕ_ρ . Note that $\text{Jord}_\rho(\sigma_{\text{cusp}})$ consists of consecutive odd numbers (starting with 1) or consecutive even numbers (starting with 2). We have

$$\sigma \hookrightarrow \prod_{\substack{\rho \\ \text{Jord}_\rho(\sigma) \neq \emptyset}} \prod_{a \in \text{Jord}_\rho(\sigma)} \delta[\rho\nu^{\frac{\phi_\rho(a)+1}{2}}, \rho\nu^{\frac{a-1}{2}}] \rtimes \sigma_{\text{cusp}}.$$

Note that this means that

$$(\sigma)_\rho \hookrightarrow \prod_{a \in \text{Jord}_\rho(\sigma)} \delta[\rho\nu^{\frac{\phi_\rho(a)+1}{2}}, \rho\nu^{\frac{a-1}{2}}] \rtimes \sigma_{\text{cusp}}. \quad (5)$$

The important thing here is that $\phi_\rho(a) \geq 0$. Assume that for certain ρ , $|\text{Jord}_\rho(\sigma_0)|$ is even, say equal to $\{a_1 < a_2 < \dots < a_{2k}\}$; let $(\sigma_0)_{\text{cusp}}$ denote the partial cuspidal support of σ_0 . This means that $\text{Jord}_\rho((\sigma_0)_{\text{cusp}})$ is empty, so that

$$(\sigma_0)_\rho \hookrightarrow \delta[\rho\nu^{-\frac{a_1-1}{2}}, \rho\nu^{\frac{a_2-1}{2}}] \times \dots \times \delta[\rho\nu^{-\frac{a_{2k-1}-1}{2}}, \rho\nu^{\frac{a_{2k}-1}{2}}] \rtimes (\sigma_0)_{\text{cusp}}. \quad (6)$$

If, on the other hand, $|\text{Jord}_\rho(\sigma_0)|$ is odd, say $\{a_1 < a_2 < \dots < a_{2k} < a_{2k+1}\}$, then there is an embedding

$$(\sigma_0)_\rho \hookrightarrow \delta[\rho\nu^{-\frac{a_1-1}{2}}, \rho\nu^{\frac{a_2-1}{2}}] \times \dots \times \delta[\rho\nu^{-\frac{a_{2k-1}-1}{2}}, \rho\nu^{\frac{a_{2k}-1}{2}}] \times \delta[\rho\nu^\alpha, \rho\nu^{\frac{a_{2k+1}-1}{2}}] \rtimes (\sigma_0)_{\text{cusp}}, \tag{7}$$

where $\alpha \in \{\frac{1}{2}, 1\}$ (cf. (2),(3)). When we examine the number of $GL(n_\rho, F)$ -blocks appearing in (5), with the number of such a blocks in (6) or (7), we see that in the latter (generic) cases, the number of such blocks is bigger and $(\sigma_0)_{\text{cusp}}$ is a representation of a group of a smaller rank than $(\sigma)_{\text{cusp}}$. This concludes our proof. ■

4. The case of tempered, non-square integrable representations and non-tempered case.

Note that in L -packet containing a tempered, non-square integrable representation, all other members of a packet are also non-square integrable (cf. Corollary 8.2 of [9]). By the (desiderata) for the properties of Langlands correspondence we know the following: let $\phi_0 : WD_F \rightarrow \hat{G}_{n_0}$ be a parameter corresponding to a generic discrete series representation (say, σ_0). For unitary groups we use $GL(N_0, \mathbb{C})$ instead of \hat{G}_{n_0} , as explained before. Let ϕ_τ be an irreducible tempered parameter for $GL(k, F)$ corresponding to an irreducible tempered representation τ of $GL(k, F)$. Let P be a standard parabolic subgroup of G_{n_0+k} with the standard Levi subgroup isomorphic to $GL(k, F) \times G_{n_0}$. Then, the L -packet Π_ϕ of G_{n_0+k} corresponding to the parameter

$$\phi = \phi_\tau + \phi_0 + \hat{\phi}_\tau$$

consists of (the isomorphism classes) of all the irreducible (tempered) constituents of the representations

$$\text{Ind}_P^{G_{n_0+k}}(\tau \otimes \sigma), \text{ where } \sigma \in \Pi_{\phi_0}.$$

Since every tempered irreducible representation is obtained in this way, the statement in the tempered case follows from the statement for the (generic) discrete case.

The case of the general (non-tempered) generic L -packet (i.e., the one containing the generic representation for our fixed character χ) also follows from the general properties of L -parameters.

So, let

$$\phi = \phi_{\tau_1}\nu^{s_1} + \dots + \phi_{\tau_r}\nu^{s_r} + \phi_0 + (\phi_{\tau_1}\nu^{s_1} + \dots + \phi_{\tau_r}\nu^{s_r})^\wedge$$

be a general generic L -packet. Here ϕ_{τ_i} , $i = 1, \dots, r$ is an irreducible tempered parameter corresponding to an irreducible tempered representation of $GL(n_{\tau_i}, F)$ and $s_1 \geq s_2 \geq \dots > s_r > 0$ and let ϕ_0 be a tempered (generic) parameter of G_{n_0} . Then, Π_ϕ consists of (the isomorphism classes) of all the Langlands quotients of standard modules

$$\tau_1\nu^{s_1} \times \tau_2\nu^{s_2} \times \dots \times \tau_r\nu^{s_r} \rtimes \pi,$$

where $\pi \in \Pi_{\phi_0}$. Again, our claim is immediate.

We note it all together:

Theorem 4.1. *Let π_0 be a generic irreducible representation of G_n belonging to an L -packet Π_ϕ . Let P_0 be a standard parabolic subgroup attached to the cuspidal support of π_0 . If $\pi \in \Pi_\phi$ has its cuspidal support attached to the standard parabolic P , then, up to the associativity, $P_0 \subset P$.*

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